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LAMINAR FLOW THROUGH UNCONSOLIDATED PACKED BEDS OF SPHERICAL AND NON-SPHERICAL MATERIAL

by

G.Malcolm Bish

Department of Mining Engineering
University of Nottingham
University Park
Nottingham
NG7 2RD
United Kingdom

ABSTRACT

This paper presents a revised form of the Kozeny equation for laminar fluid flow through packed beds in terms of particle shape factor and surface mean diameter, rather than specific surface. The equation is based upon a model of pore formation which takes into account the interlocking of irregular particles. It is concluded that for comparative purposes the 'effective' specific surface is a quantity which can be derived exactly from permeability and porosity measurement. The effective specific surface affords a ready means of control in comparing one packing with another, provided that a standard, conventional method is adopted for the assessment of surface mean diameter.

INTRODUCTION

For well over forty years the accepted equation of fluid flow in permeametry in the laminar range has been the Kozeny-Carman(1,2) equation.

Carman(3) and Dallavalle(4) suggested independently in 1938 that the determination of specific surface should be carried out by means of permeability measurement, and by 1941 Carman had elaborated upon the appropriate methods for doing so. The importance of Kozeny-Carman in permeametry has been recognized in the literature, and in International Symposia on Particle Size Analysis, since that time.

The drawbacks of the Kozeny-Carman equation have been appreciated from an early stage and discussed in a number of texts. One significant problem has been the failure of the equation to deal properly with flow through material departing radically from the spherical or near-spherical. It therefore fails to make accurate predictions of permeability for materials subject to interlocking rather than point contact. Another problem is Kozeny's use of the specific surface to describe material size. Clearly, it is possible for materials of the same 'diameter' to have differing values of specific surface.

In this paper, shape and diameter are separately identified, and hydraulic radius is developed with especial reference to the interlocking of particles. It is considered that the revised equation of flow worked out represents a more general and a more accurate description of the factors affecting hydraulic conductivity, and makes a clearer statement of the nature of flow in the laminar range. The most interesting conclusion to be drawn from the analysis is not only to confirm that the Kozeny constant is merely a consequence of the nature of the packing of the material, but also that it varies between different materials at the same porosity, even where they have identical shape factors and diameters and, therefore, identical specific surfaces. This makes prediction of the constant for particular porosity values in a given material an impossible task; the value of the constant is unique to the shape of the material and its own unique mode of packing. For this reason, it is a conclusion of the thesis(5) in which the new equation is derived that the determination of surface area for quality control purposes would be more soundly based on the 'effective' specific surface, as defined in this paper.

The new equation proposed is analogous to the Kozeny equation but is considered to be more general in its concept and in its application. The new equation offers an explanation of the mechanism of flow through spherical and non-spherical particles, and suggests a theoretical basis for the approximate value of the Kozeny constant in spherical material over the range of porosities investigated.

THE UNIT CELL AND POROSITY

Introduction

Equations so far developed to describe flow through packed porous beds do not include an effective parameter to account for the shape of the particle: the analysis which now follows results in an equation which takes the effect of particle shape into account as an integral part of the initial model of flow devised.

Unit cell

The failure of Slichter's(6) analysis diverted attention from his basic approach. In any investigation of the spatial relationships

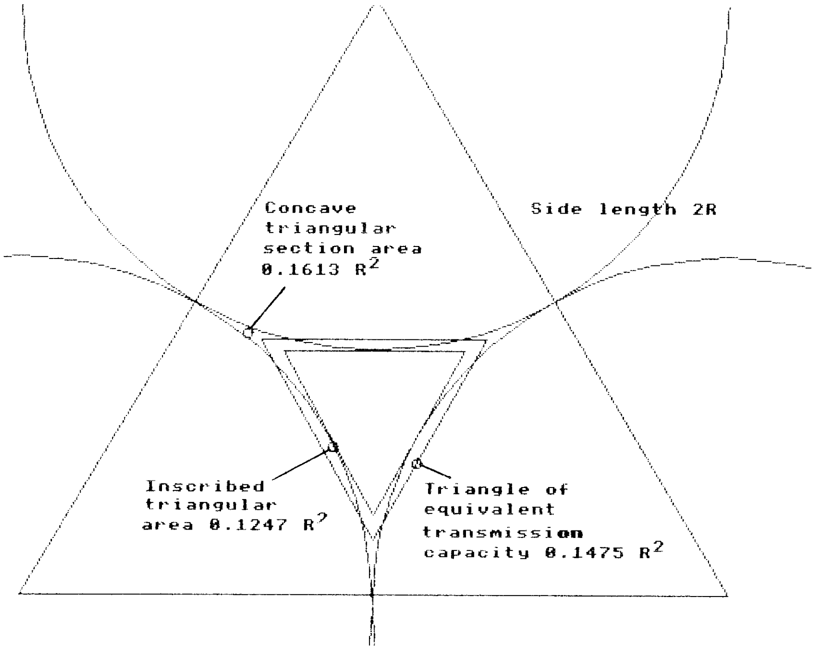

Figure 1 Slichter minimum pore cross section

of particles within a packed bed, however, it is still appropriate to start where he started - with the unit cell based on spherical arrays. Fig.1 shows the typical pore cross section investigated by Slichter. Clearly, the cross-section shape of pore tube to be observed in this diagram is that at the plane of minimum pore cross-section, called by Graton and Fraser(7) the throat plane. To describe the packings, for flow assumed to take place normal to the cross-section of the throat plane (at the throat planes themselves), the ratios now described below are employed.

Volume porosity

It is possible to obtain the volume of the unit cell in terms of an equivalent sphere diameter. Let the diameter of the equivalent sphere be 'a'. The unit-cell volume is c_1a^3 where c_1 is a coefficient varying according to the geometry of the cell. Following Slichter, and Graton and Fraser, it is clear that each cell, of whatever shape, must contain parts of a sphere which all together make up a unit sphere. Therefore, the total volume of the spheres contained in unit volume of the packing must be

$$\frac{1}{c_1a^3} \times \frac{\pi.a^3}{6} = \frac{\pi}{6c_1} \qquad (1)$$

and this quantity must equal 1-n where n is the porosity, so that

$$n = 1 - \frac{\pi}{6c_1} \qquad (2)$$

for packings of spheres.

Area porosity

The area porosity n' is defined for any plane through the unit cell as the ratio of the pore area to the area of cross-section of the cell at that plane.

Porosity ratio

The porosity ratio is defined as the ratio of the area porosity for any chosen plane through the unit cell to the volume porosity

$$\tau = \frac{n'}{n} \qquad (3)$$

Tortuosity

In all cases, the thickness L is the distance between two faces of the unit cell and perpendicular to them. The tortuous length L_e is considered to be the distance, inside the unit cell, along the locus of the centroid of the pore cross-section between one throat plane and the next. The path described in this way by the centroid is assumed to conform with the shape of the surrounding spheres and to follow the shortest possible tortuous route from throat plane to succeeding throat plane.

AN ORIGINAL ANALYSIS OF FLOW

An idealized model packing

It is argued that flow must be conditioned by the shape and area of the cross-section at the throat plane. Porosity is represented by equating the number of imaginary spheres of diameter 'a' in an idealized model packing to the number of irregular particles actually present in the packing. For unit volume of the packing, the following expression is obtained

$$\frac{1}{c_1 a^3} = \frac{1-n}{v(\overline{qx})^3} \qquad (4)$$

from which

$$a^3 = \frac{v}{c_1(1-n)} \cdot (\overline{qx})^3 \qquad (5)$$

where v and $(\overline{qx})$ are the Heywood(8) volume coefficient, and the statistical mean volume diameter, respectively.

In equation (4), the number of spheres is $1/(c_1 a^3)$. Clearly, the same number can be obtained either by maintaining c_1 constant and varying 'a', or by maintaining 'a' constant and varying c_1, which is what was done in the previous section on spherical porosity. From now on, it is more convenient to represent any packed-bed material by a unit cell based on a constant value of c_1. On this basis, the concept underlying equation (5) is of 'a' as an arbitrary diameter of an imaginary sphere defined in terms of the mean volume diameter of the particles in the bed, and varying in accordance with equation (5) to describe porosity change. In equation (5), c_1, v and $(\overline{qx})$ are constant for a given material and c_1 is constant for all materials.

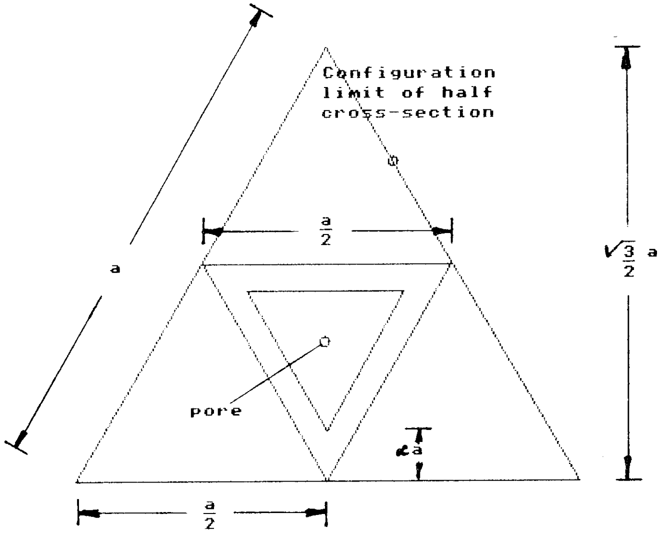

Figure 2 Idealized representation of interlocking for irregular particles

Description of idealized cross section of pore

The hydraulic radius can also be expressed in terms of 'a' and n. In order to justify a relationship, consider a unit-cell section based on the throat-plane cross-section area shown in Fig.2. The section is idealized to represent the average cross-section which is regarded as being continuous throughout the bed. In order to account for a variation in porosity in non-spherical material, the imaginary model spheres of diameter 'a' are considered to be interlocked to a certain extent, the common length between contiguous particles being $2.\alpha.a$. For such a section, the argument then runs as follows

height half unit cell $= \frac{\sqrt{3}}{2} a$ (6)

half unit-cell cross-section area $= \frac{\sqrt{3}}{4} a^2$ (7)

side length of pore $= \frac{a}{2} - 2 \frac{\sqrt{3}}{2} \alpha a$

$= \left[\frac{1}{2} - \alpha\sqrt{3} \right] a$ (8)

pore perimeter $= 3 \left[\frac{1}{2} - \alpha\sqrt{3} \right] a$ (9)

height of pore area $= \frac{\sqrt{3}}{2} . \frac{a}{2} - \alpha.a - \frac{\alpha.a}{2}$

$= \frac{\sqrt{3}}{2} \left[\frac{1}{2} - \alpha\sqrt{3} \right] a$ (10)

pore area $= \frac{1}{2} \left[\frac{1}{2} - \alpha\sqrt{3} \right] a \frac{\sqrt{3}}{2} \left[\frac{1}{2} - \alpha\sqrt{3} \right] a$

$= \frac{\sqrt{3}}{4} \left[\frac{1}{2} - \alpha\sqrt{3} \right]^2 a^2$ (11)

and this area must be given by

$$\tau . n . \frac{\sqrt{3}}{4} . a^2$$

so that

$$\left[\frac{1}{2} - \alpha\sqrt{3} \right]^2 = \tau . n \quad (12)$$

Analysis of hydraulic radius

The hydraulic radius is

$$m = \frac{\frac{\sqrt{3}}{4}\left[\frac{1}{2} - \alpha/3\right]^2 a^2}{3\left[\frac{1}{2} - \alpha/3\right] a}$$

$$= \frac{1}{4\sqrt{3}}\left[\frac{1}{2} - \alpha/3\right] a \qquad (13)$$

and

$$m^2 = \frac{1}{48}\left[\frac{1}{2} - \alpha/3\right]^2 a^2$$

$$= \frac{1}{48} \cdot \tau \cdot n \cdot a^2$$

$$= c_2 \cdot \tau \cdot n \cdot a^2 \qquad (14)$$

where c_2 is a constant having the value 1/48.

From equation (5), it can be seen that a reduction in porosity, n, results in a reduction in 'a'. This means that relatively lower porosities are represented in the model by smaller spheres packing together. It seems reasonable to suppose that the three-dimensional mechanism involved is such that the porosity ratio itself is proportional to porosity, that is to say

$$\tau = \text{constant} \cdot n \qquad (15)$$

and that, therefore, pore area is a function of the square of the porosity. If c_3 is the constant in equation (15), then

$$m^2 = c_2 \cdot c_3 \cdot n^2 \cdot a^2$$

$$= \phi(n^2 . a^2) \qquad (16)$$

The same kind of relationship can be demonstrated for the Graton and Fraser Case 1 maximum concave-square cross-section, and since these sections are extreme forms it seems reasonable to take equation (16) as having general application with c_2 and c_3 varying with average cross-section shape, but assumed constant for any one packed-bed material.

From equation (5),

$$a^2 = \frac{1}{(c_1)^{2/3}} \cdot v^{2/3} \cdot \frac{1}{(1-n)^{2/3}} \cdot (q\bar{x})^2 \qquad (17)$$

and if this value of a^2 is substituted in equation (16)

$$m^2 = \left[\frac{c_2 . c_3}{(c_1)^{2/3}}\right] \cdot v^{2/3} \cdot \frac{n^2}{(1-n)^{2/3}} \cdot (q\bar{x})^2 \qquad (18)$$

AN ORIGINAL EQUATION OF FLOW

Substitution of hydraulic radius
in basic capillary-tube equation

Kozeny showed that the actual velocity through the pores u_e must be given by

$$u_e = \frac{u}{n} \cdot \frac{L_e}{L} \qquad (19)$$

where u is the average velocity of flow and, from Poiseuille(9)

$$u_e = \frac{1}{k_o} \cdot \frac{g}{\nu} \cdot m^2 \cdot \frac{H}{L_e} \qquad (20)$$

where k_o is a constant defining pore shape, g the acceleration caused by force of gravity, ν the kinematic viscosity, and H the resultant driving head across the packed bed. Following Kozeny, it is now possible to expand equation (20) by substituting for m^2 from equation (18) to obtain

$$u_e = \left[\frac{c_2 . c_3}{(c_1)^{2/3}}\right] \cdot \frac{1}{k_o} \cdot \frac{g}{\nu} \cdot v^{2/3} \cdot \frac{n^2}{(1-n)^{2/3}} \cdot (q\bar{x})^2 \cdot \frac{H}{L_e} \qquad (21)$$

In equation (19), Kozeny assumed that the area porosity equalled the volume porosity; that is, he assumed that the pore area took up all the available volume porosity n (thought of here as an area porosity). However, in the argument now being pursued, the area porosity n' must be substituted for the volume porosity n in the Kozeny statement, so that

$$u_e = \frac{u}{n'} \cdot \frac{L_e}{L}$$

$$= \frac{1}{c_3} \cdot \frac{u}{n^2} \cdot \frac{L_e}{L} \qquad (22)$$

If the right-hand side of equation (22) is substituted in equation (21), then

$$u = \left[\frac{c_2 . (c_3)^2}{(c_1)^{2/3}}\right] \cdot \frac{1}{\left[k_o \left[\frac{L_e}{L}\right]^2\right]} \cdot \frac{g}{v} \cdot v^{2/3} \cdot \frac{n^4}{(1-n)^{2/3}} \cdot (q\bar{x})^2 . i$$

$$= \left[\frac{c_o}{k_1}\right] \cdot \frac{g}{v} \cdot v^{2/3} \cdot \frac{n^4}{(1-n)^{2/3}} \cdot (q\bar{x})^2 \cdot i \qquad (23)$$

where k_1 is the Kozeny constant and c_o is a coefficient

$$\frac{c_2 . (c_3)^2}{(c_1)^{2/3}}$$

which varies with particle shape and is constant for porosity change in the same material. From equation (23)

$$\frac{kv}{g} = \left[\frac{c_o}{k_1}\right] \cdot v^{2/3} \cdot \frac{n^4}{(1-n)^{2/3}} \cdot (q\bar{x})^2 \qquad (24)$$

where k is the coefficient of effective permeability derived from Darcy's(10) law.

Comparison with Kozeny equation

If the Kozeny equation is written in terms of kv/g, then

$$\frac{k\upsilon}{g} = \frac{1}{k_1} \cdot \left[\frac{v}{f}\right]^2 \cdot \frac{n^3}{(1-n)^2} \cdot x_s^2 \qquad (25)$$

Values of (c_o/k_1) can be derived by means of equation (24) and values of $(1/k_1)$ from equation (25) for the same given shapes and porosities. From the two equations

$$c_o = \left[\frac{v^{2/3}}{f}\right]^2 \cdot \frac{1}{n.(1-n)^{4/3}} \cdot \left[\frac{x_s}{qx}\right]^2 \qquad (26)$$

and if c_o is substituted in equation (24) according to equation (26) then

$$\frac{k\upsilon}{g} = \left[\frac{1}{k_1} \cdot \frac{1}{n(1-n)^{4/3}}\right] \cdot \left[\frac{f}{v}\right]^{-2} \cdot \frac{n^4}{(1-n)^{2/3}} \cdot x_s^2 \qquad (27)$$

which is to equate the equation (24) to the Kozeny equation, with the shape factor quoted in reciprocal form for ease of calculation. The resulting general expression can be stated as

$$\frac{k\upsilon}{g} = \left[\frac{\phi(n_1)}{k_1}\right] \cdot \left[\frac{f}{v}\right]^{-2} \cdot \phi(n) \cdot x_s^2 \qquad (28)$$

where

$$\phi(n_1) = \frac{1}{n.(1-n)^{4/3}} \quad \text{and} \quad \phi(n) = \frac{n^4}{(1-n)^{2/3}}$$

It can be demonstrated that $\phi(n_1)$ is sensibly constant over a large range in porosity. It has a value which is exactly 4.94 at a porosity of 0.400, while being within 1% of this value for a range of porosity from 0.375 to 0.480, and still within 5% for a range of porosity from 0.330 to 0.530. In other words, over the range of porosity regarded as applicable to formations of natural sands, $\phi(n_1)$ can be regarded as constant.

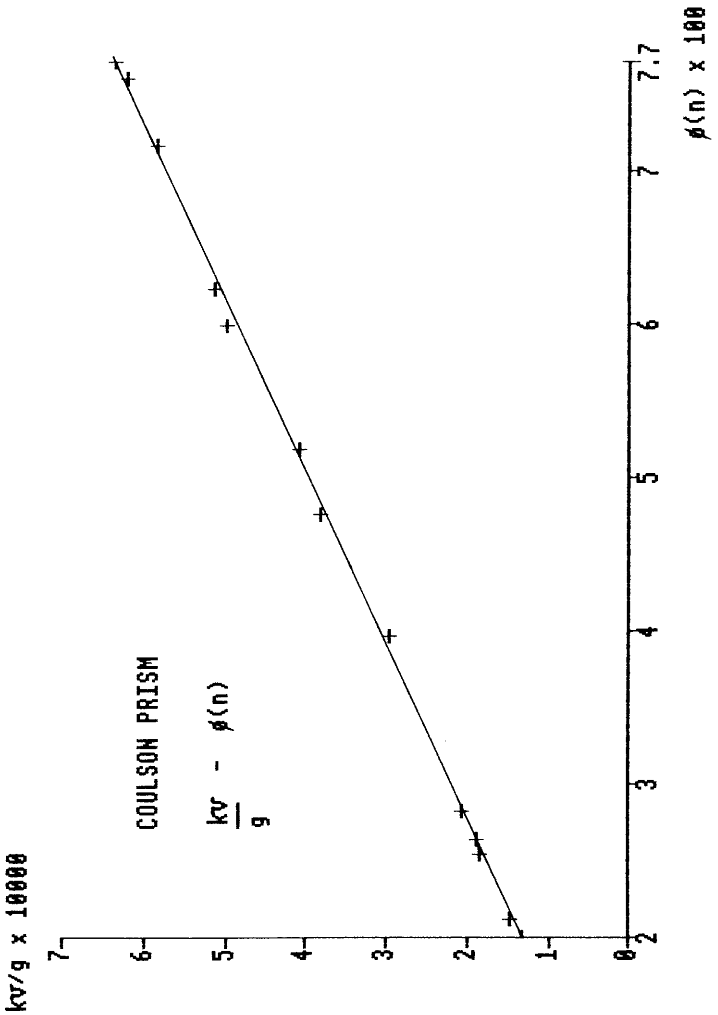

Figure 3 Plot of $\frac{kv}{g}$ - $\frac{n^4}{(1-n)^{2/3}}$

CONFIRMATION OF ANALYSIS BY TESTING ON PUBLISHED DATA

Both Coulson(11) and Wyllie and Gregory(12) have published permeability and porosity data for beds of regular particles of known shape. The porosity function $n^4/(1-n)^{2/3}$ can be tested for the Coulson data by plotting kv/g as ordinate against $n^4/(1-n)^{2/3}$ as abscissa. In general, the plots, an example of which is shown in Fig.3, are straight lines. This relationship is further corroborated by plots of the Wyllie and Gregory data with similar results.

Original equation and x_s^2

The next stage in verification is to show that kv/g is proportional to the square of the particle size. To do this, it is necessary to define a quantity A which is the $\frac{kv}{g}$-value for unit porosity function. For if

$$A = \frac{kv}{g} \cdot \frac{1}{\phi(n)} \tag{29}$$

then

$$A = \left[\frac{\phi(n_1)}{k_1} \cdot \left[\frac{f}{v}\right]^{-2} \right] \cdot x_s^2 \tag{30}$$

In considering the right-hand side of this expression, it can be seen that, for material of the same shape, a cube for example, the material will have a constant shape factor $(f/v)^{-2}$, although it may have different size fractions with diameters x_s varying with size. The function $\phi(n_1)$ is sensibly constant and it will be assumed for the purpose of this exercise that <u>k_1 can be regarded as constant for a range of diameter squared x_s^2 in a material of the same shape and all at the same porosity</u>. The foregoing has been underlined to emphasize the importance of the conditions under which the test must be applied.

Then, for constant $\phi(n_1)$, $(f/v)^{-2}$ and k_1

$$A \propto x_s^2 \tag{31}$$

and straight-line plots result from plots of $A - x_s^2$ for materials of the same shape at the same porosity. The surface mean diameter x_s is assessed by the methods of Heywood, using the equations

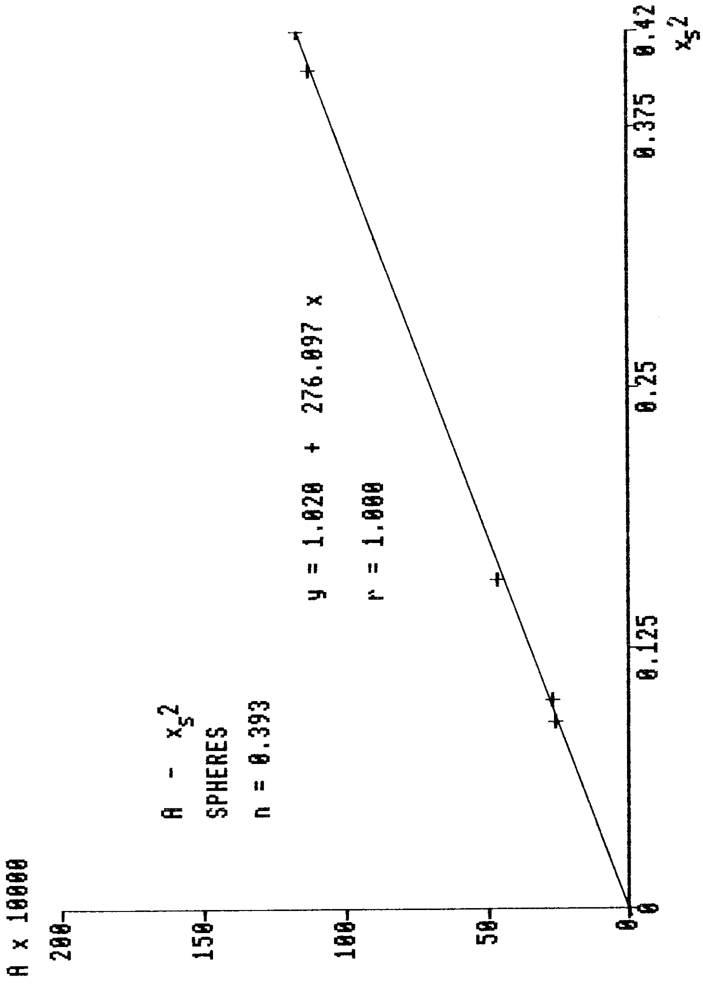

Figure 4 Plot of $A - x_s^2$ for spheres

derived by Hatch and Choate(13).

Spheres present a little difficulty in that, although $(f/v)^{-2}$-values are common, porosity variation is limited in the Coulson series. Only single porosities were investigated in three sizes and only two porosities in the other two. The Wyllie and Gregory series for a single sphere is better with four values giving excellent correlation for the $(kv/g) - \phi(n)$ plot. This makes it possible to compare two sizes of Coulson sphere at a porosity of 0.393, for kv/g values actually observed, with one derived value of kv/g from Coulson and one from Wyllie and Gregory at that porosity using the appropriate regression equations from plots of $kv/g - n$. For the values detailed in Table 1, which have been calculated for a porosity of 0.393, A as ordinate is plotted

TABLE 1

VALUES OF A AND x_s^2 FOR SPHERES

$n = 0.393 \quad \phi(n) = 0.03327$

Sphere	$\frac{kv}{g}$ x10000	A x10000	x_s	x_s^2
Coulson 2	3.72907	112.09	0.635	0.40323
Coulson 3	1.52954	45.97	0.397	0.15761
Coulson 4	0.90843	27.30	0.317	0.10049
Wyllie and Gregory	0.87040	26.16	0.300	0.09000

against x_s^2 as abscissa in Fig.4. The regression is shown on the figure. There is a high degree of correlation.

Now, if the sphere plot, using the same data detailed in Table 1, is forced through the origin as theory demands, the result is most interesting. The plot of Fig.4 shows no change, but the regression is changed slightly. Now, the slope of the plot through the origin is 0.027775, which is exactly the value of $(f/v)^{-2}$ for a sphere. Therefore, from equation (30)

$$k_1 = \phi(n_1) \times \frac{(f/v)^{-2}}{\text{slope of plot}}$$

$$= \phi(n_1) \times \frac{0.027777}{0.027775}$$

that is to say, k_1 is demonstrably equal to $\phi(n_1)$, as theory demands should be the case for a spherical particle. This is a highly encouraging result and explains why so many workers have adopted a value of the order of 4.94 for k_1 in spherical arrays at or close to a porosity of 0.400.

Similar results are obtained for a series of plots at different porosity values for other shapes. The evidence is that for whatever shape there is a linear relationship through the origin of the form

$$A = \text{constant} \; . \; x_s^2 \qquad (32)$$

where the constant is

$$\frac{\phi(n_1)}{k_1} \; . \; \left[\frac{f}{v}\right]^{-2}$$

and this, of course, is fully in accord with hydraulic-radius theory, in which permeability is regarded as being proportional to the reciprocal square of the specific surface.

Permeability and the shape factor

The final stage in verification is to relate permeability to the shape factor for the material being investigated. To do this, it is necessary to define a quantity A_o which is the A-value for unit diameter-squared. For, if

$$A_o = A \; . \; \frac{1}{x_s^2} \qquad (33)$$

then

$$A_o = \frac{\phi(n_1)}{k_1} \; . \; \left[\frac{f}{v}\right]^{-2} \qquad (34)$$

and

$$A_o \; . \; k_1 = \phi(n_1) \; . \; (f/v)^{-2} \qquad (35)$$

If A_ok_1 is plotted as ordinate against $(f/v)^{-2}$ as abscissa, then the slope of the plot must be given by the $\phi(n_1)$-value for the particular porosity value being investigated.

Again using values calculated from the Coulson, and Wyllie and Gregory, data, $A_o k_1$ and $(f/v)^{-2}$ have been plotted with $A_o k_1$ as ordinate and $(f/v)^{-2}$ as abscissa for a single porosity value in Fig.5. Similar results are obtained throughout a series of plots over a range of porosity from 0.32 to 0.50. For each value of porosity there is a linear relationship through the origin of the form

$$A_o \cdot k_1 = \text{constant} \cdot (f/v)^{-2} \qquad (36)$$

where the constant is the appropriate value of $\phi(n_1)$ for the porosity involved.

FLOW MECHANISM AND SPECIFIC SURFACE

Nature of flow mechanism

The interesting thing to be noted from the model of flow now fully defined is that, for any particular porosity value, a range of k_1 values will satisfy the framework of relationships established for $A - x_s^2$ and $A_o.k_1 - (f/v)^{-2}$ plots. For each porosity value in respect of a given shape, $A_o.k_1$ is easily determined. It is the value of the porosity function $\phi(n_1)$ multiplied by the reciprocal square of the shape factor for the particle. But A_o and k_1 are interdependent and A_o will assume different values for differing k_1, while preserving an identical value for $A_o.k_1$ which will remain the same as k_1 varies.

So, it is possible to think in terms of two or more particles having identical shape factors and the same diameter, therefore the same specific surface, but with different k_1 values for the same porosity in each material, and consequently differing permeabilities for that same porosity. It seems, in theory, that even though materials can have the same specific surface while having slightly different shapes, the slightest variation in shape causes the materials to pack together in ways which create different packing formations so that the configuration of the pores is not the same. This difference in configuration is reflected in the variation of k_1-values capable of satisfying all the basic relationships defined. In these circumstances, it would be most unlikely that it could be possible to predict in advance the value of k_1 applying to a particular shape of unknown shape factor for a given porosity.

It can be said, then, that it is possible to demonstrate for known shapes the relationships which must obtain between the parameters of the main equation (28), but that no unique relationship seems attainable for the estimation of a k_1 value applicable to a given particular shape.

The evidence from an examination of the data provided by Coulson, and Wyllie and Gregory, for regular particles clearly supports a theoretical model framework which can be summarized as follows:-

1 Main equation:

$$\frac{k\upsilon}{g} = \frac{\phi(n_1)}{k_1} \cdot \left[\frac{f}{v}\right]^{-2} \cdot \phi(n) \cdot x_s^2 \qquad (37)$$

2 For the same shape at the same porosity:

$$\frac{k\upsilon}{g} \cdot \frac{1}{\phi(n)} = A = \left[\frac{\phi(n_1)}{k_1} \cdot \left[\frac{f}{v}\right]^{-2}\right] \cdot x_s^2 \qquad (38)$$

3 For unit diameter of material:

$$A \cdot \frac{1}{x_s^2} = A_0 = \frac{\phi(n_1)}{k_1} \cdot \left[\frac{f}{v}\right]^{-2} \qquad (39)$$

and

$$A_0 \cdot k_1 = \phi(n_1) \cdot \left[\frac{f}{v}\right]^{-2} \qquad (40)$$

with plots of $A_0.k_1 - (f/v)^{-2}$ through the origin. This relationship holds for the whole range of shapes and for the whole range of porosities exhibited by single-value or multi-value plots.

So, we may take the theory and the range of equations (37) to (40) as an accurate expression of the mechanism of flow through an extended range of shapes and sizes of packed-bed material over an extended range of porosity.

The $k_1/\phi(n_1)$ ratio

The Kozeny constant k_1 is the numerator of the square of what may be termed an interlocking factor, and the denominator is $\phi(n_1)$. In the case of a sphere, it has been clearly demonstrated that $k_1 = \phi(n_1) = 4.94$ over a limited range, and so $k_1/\phi(n_1)$ is unity. In which case

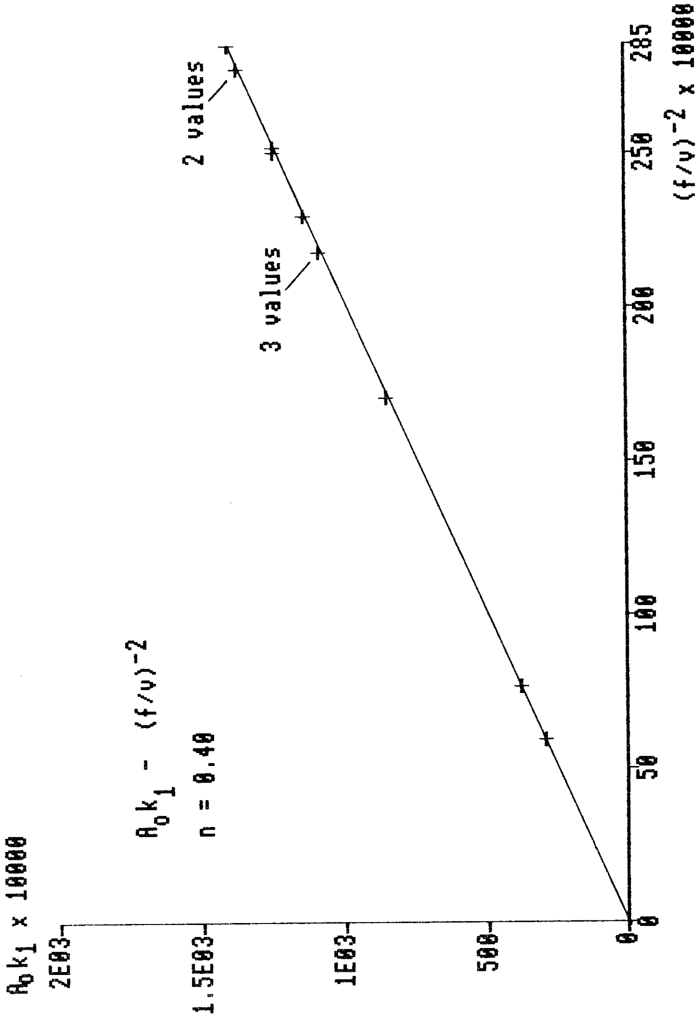

Figure 5 Plot of $A_0k_1 - (f/v)^{-2}$ for various shapes

$$\frac{kv}{\varepsilon} = \left[\frac{f}{v}\right]^{-2} . \phi(n) . x_s^2 = \frac{1}{S_o^2} . \phi(n) \qquad (41)$$

and this is an expression which can be taken to hold for a sphere which is equivalent to a particle with a shape factor f/v and a surface mean diameter x_s, but which is not spherical in shape. This 'effective' S_o will be referred to as $S_{o\,\text{effective}}$

From equation (41)

$$S_{o\,\text{effective}}^2 = \frac{\phi(n)}{\left[\frac{kv}{\varepsilon}\right]} = A^{-1} \qquad (42)$$

and so

$$S_{o\,\text{effective}} = A^{-0.5} \qquad (43)$$

The diameter of the effective sphere must be

$$x_{s\,\text{effective}} = \frac{6}{A^{-0.5}} \qquad (44)$$

Again, from equations (39) and (41), in which x_s is the surface mean diameter of the actual particle

$$A_o = \frac{A}{x_s^2}_{\text{act}} = \left[\frac{f}{v}\right]^{-2}_{\text{eff}} = \text{slope of } A - x_s^2 \text{ plot} \qquad (45)$$

and since, from equations (39) and (45)

$$\frac{\phi(n_1)}{k_1} . \left[\frac{f}{v}\right]^{-2}_{\text{act}} = A_o = \left[\frac{f}{v}\right]^{-2}_{\text{eff}} \qquad (46)$$

then

$$\frac{k_1}{\phi(n_1)} = \frac{(f/v)^{-2}_{act}}{(f/v)^{-2}_{eff}} = \frac{(f/v)^2_{eff}}{(f/v)^2_{act}} = \frac{(f/v)^2_{eff}\, x_{s\,act}^2}{(f/v)^2_{act}\, x_{s\,act}^2} = \frac{S_{o\,eff}^2}{S_{o\,act}^2} \qquad (47)$$

from which it can be seen that $k_1/\phi(n_1)$ is the ratio which defines the effective surface presented to flow for any actual particle surface by means of

$$S_{o\,eff}^2 = \frac{k_1}{\phi(n_1)} \cdot S_{o\,act}^2 \qquad (48)$$

CONCLUSIONS

In spherical or near-spherical sands, k_1 is equal to $\phi(n_1)$, that is $k_1 = \phi(n_1) = 4.94$. That has been confirmed in this present analysis and is a result of the fact that spheres pack together with an absence of interlocking. Strictly speaking, the value of the $\phi(n_1)$ porosity function will be 4.94 only at a porosity of 0.400, but it remains approximately equal to 4.94 over an extended range of porosity either side of 0.400. That is why within the range of porosities applicable to spherical packings a Kozeny constant value of $\phi(n_1)$ would be the most appropriate assumption in the Kozeny equation

Historically, the interest in trying to establish k_1 values generally for varying particle shape has been to estimate in turn the permeabilities for given porosities and absolute values of the actual specific surface $S_{o\,actual}$ of the packed-bed material. However, it does appear that there is no practicable way to determine k_1 values for particles of irregular shape and unknown shape factor. This is because packings of particles having the same specific surface and at the same porosity nevertheless exhibit different values of permeability. The permeability of a packing is unique to the particular shape of of the constituent particles and the pore spaces they create, and it seems unlikely that any way can be found to predict the Kozeny constant in advance for any packing material of unknown shape factor.

Given that x_s can be assessed from image analysis or by other means, then since effective specific surface $A^{-0.5}$ is easily determined for known permeability and porosity, effective specific surface can be used as a control for comparative purposes, rather than the more problematical actual specific surface which is so difficult to estimate for irregular particles. The effective specific surface can be determined exactly from permeability testing and offers an entirely accurate means of comparison between different packings.

The use of effective specific surface means that the surface mean diameter must be assessed as accurately as possible and by means of a standard, conventional method of calculation, if comparison is to be possible between the results of various workers in this field. Work done by the author(5) rested upon the Heywood approach to particle measurement and employed the Hatch and Choate equations to determine statistical average diameters. This procedure, or an equivalent standard procedure, must be used to ensure true comparability.

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LIST OF NOTATIONS
(in order of appearance)

		Dimensions
a	diameter of equivalent sphere in idealized model packing	m
c_1	coefficient for unit cell volume	n.d
π	pi	n.d
n	volume porosity	n.d
n'	area porosity	n.d
τ	porosity ratio $\left[= \frac{n'}{n} \right]$	n.d
L	thickness of bed	m
L_e	actual length of pore	m
v	Heywood volume coefficient	n.d
$\bar{q}x$	statistical mean volume (mass) diameter	m

$2.\alpha$	extent of interlocking between non-spherical particles	n.d
m	hydraulic radius	m
c_2	constant in equation $m^2 = c_2.\tau.n.a^2$	n.d
c_3	constant in equation $\tau = c_3.n$	n.d
u_e	actual velocity through pore	$\frac{m}{sec}$
u	average velocity of flow	$\frac{m}{sec}$
k_0	constant defining pore shape	n.d
g	acceleration caused by force of gravity	$\frac{m}{sec^2}$
ν	kinematic viscosity of fluid (stoke) $\left[= \frac{\mu}{\rho} \right]$	$\frac{m^2}{sec}$
μ	viscosity of fluid (poise)	$\frac{kg}{m.sec}$
ρ	density of fluid	$\frac{kg}{m^3}$
H	resultant driving head across bed thickness	m
$\left[\frac{L_e}{L}\right]^2$	tortuosity	n.d
i	hydraulic gradient (= H/L)	n.d
c_0	overall shape coefficient $\left[= \frac{c_2(c_3)^2}{(c_1)^{2/3}} \right]$	n.d
k_1	Kozeny constant $\left[k_0\left[\frac{L_e}{L}\right]^2 \right]$	n.d
k	Darcy coefficient of effective permeability (= u/i)	$\frac{m}{sec}$
f	Heywood surface coefficient	n.d
x_s	statistical surface mean diameter ($= \sum x^3 / \sum x^2$) (Heywood)	m
$\sum$	summation symbol	
x	statistical diameter of particle in a particulate system	m

$\left[\frac{f}{v}\right]$	shape factor (Heywood)	n.d
ø	denotes 'function of'	
$ø(n_1)$	porosity function (= 4.94)	n.d
ø(n)	porosity function expressing effect of change in porosity	n.d
A	(kv/g)-value for unit porosity function	m^2
A_o	A-value for unit diameter-squared	n.d
$S_{o\,eff}$	specific surface of 'effective' sphere	m^{-1}
$S_{o\,act}$	specific surface of actual non-spherical particle	m^{-1}

REFERENCES

(1) KOZENY, J.S., Über kapillare leitung des wassers im boden, Sitzb.Akad.Wiss.Wein.Math-naturw.Kl., 136(Abt.IIa), 271-306, 1927

(2) CARMAN, P.C., Fluid flow through granular beds, Trans.Inst.Chem.Eng., 15, 150, 1937

(3) CARMAN, P.C., The determination of the specific surface of powders. I, J.Soc.Chem.Ind., 57, 225, 1938

(4) DALLAVALLE, J.M., Surface area in packed columns, Chem.& Met.Eng., 45, 688-691, 1938

(5) BISH, G.M., Laminar fluid flow through unconsolidated beds of spherical and non-spherical particles, Ph.D.Thesis, University of Nottingham, 1987

(6) SLICHTER, C.S., Theoretical investigation of the motion of ground waters, U.S.Geol.Surv., 19th Ann.Rept., Pt.II, 301, 1898

(7) GRATON, L.C., and FRASER, H.J., Systematic packing of spheres; with particular relation to porosity and permeability, Journal of Geology, xliii, 785, 1935

(8) HEYWOOD, H., Calculation of the specific surface of a powder, Proc.Inst.Mech.Eng., 125, 383, 1933

(9) POISEUILLE, J., Recherches expérimentales sur le mouvement des liquides dans les tubes de trés-petit diamètre, Inst. de France, Acad. de Sci., Mémoires présentés par divers savants, 9, 433-543, 1846

(10) DARCY, H.G.P., Les fontaines publique de la ville de Dijon Victor Dalmont, Paris, 1856

(11) COULSON, J.M., The flow of fluids through granular beds: effect of particle shape and voids in streamline flow, Trans. Inst. Chem. Eng., 27, 237, 1949

(12) WYLLIE, M.R.J., and GREGORY, A.R., Effect of porosity and particle shape on Kozeny-Carman constants, Ind. Eng. Chem., 47, 1379, 1955

(13) HATCH. T., and CHOATE. S.P., Statistical description of the size properties of non-uniform particulate substances, J. Franklin Inst., 207, 369, 1929