

DECISION MODELS FOR BAUXITE MINING  
UNDER WATER HAZARD

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SUMMARY

Development of underground bauxite mining under karstic water hazard is being planned in the Transdanubian region of Hungary. The karstic aquifer also serves for regional water supply and provides natural recharge for several thermal springs in the region. A mining policy resulting in a best compromise among the three conflicting goals of water hazard alleviation, water supply and thermal springs protection is sought. Alternative engineering systems include the combination of mine dewatering, artificial recharge and decreasing local transmissivity by grouting. The systems model consists of a finite difference solution of the partial differential equation describing regional karstic water flow.

Problem description

The general problem considered consists of the planning and operation of regional industrial development with due regard to environmental protection. This regional development has at least two sets of objectives: 1/ economic, and 2/ environmental objectives. Social and political objectives can also be accounted for but it is assumed that the economic and environmental objectives are predominant.

Economic objectives are measured by the common indicators of economic efficiency such as cost, net benefit, cost-benefit ratio, and are expressed in monetary units. The environmental objectives refer mostly to the enhancement of the quality of land, water and air and are generally expressed in natural units such as the amount of natural resources or pollutants.

In order to define and solve the problem, information on at least two sub-systems is necessary:

- I. the industrial sub-system is composed of engineering-economic elements such as capacity, location and technology of plants.
- II. the environmental sub-system is composed of physical elements such as the quality of land, water and air modified by the industrial development.

In most of the cases the two sets of objectives are in discord. The classical approach is to fix one of the sets /e.g. by environmental standards/ and optimize the system according to the other set.

The other approach is to consider both sets simultaneously and to find a satisfactum [1] instead of an optimal solution.

In the Bakony region of 10.000 km<sup>2</sup> in the Transdanubian Mountain in Hungary /Fig. 1/ large-scale bauxite deposits are being planned to extract.

There are several existing and planned mining sites for surface or underground operation. A typical vertical distribution of bauxite is illustrated in Fig. 2. Bauxite is mined for processing within and outside the region and for export.

In many mining sites the regional karstic water level is higher than the bauxite deposit. There are two main possibilities /and the combinations thereof/ of mining under water hazard in the region: water level lowering /mine dewatering/ and the decreasing of local transmissivity by grouting.

Given the available capacity of processing plants and the committed amount of export for a fairly long time ahead /15-20 years/, the economic objective is to allocate production rates among existing and candidate mine under minimum discounted cost.

Among environmental factors /land, water and air/, the impact of bauxite mining on the regional groundwater system is of the greatest concern. The groundwater system is based on a large-scale karstic aquifer of several hundred meters deep, and is being fed by infiltrated rainwater and inflow from rivers such as the Danube [2].

In its original state some 30 years ago, outflow from the aquifer was represented by a number of springs, water-supply wells, thermal baths and supply for adjacent groundwater systems. As a result of mine water withdrawals for bauxite mining operations the original state of the system has changed: a/ the regional karstic water level has drop-

ped, and b/ the flow of several springs and thermal baths have stopped or decreased.

At present, there is a major environmental objective to maintain at least the present state of the karstic water system. In fact, the most important thermal baths such as Héviz and water wells for cities such as Veszprém are still safely operating. Two questions arise: 1/ how to characterize the environmental state, and 2/ how to fix the numerical values representing state. The environmental state may correspond to the flow and quality of springs, thermal baths and water wells in the region. There is, however, some contradiction about the numerical values of these flows and quality parameters, representing a sound regional environment. Due to this contradiction, we would not consider fixed values but regard the case either as:

- a. a multiobjective problem where the minimization of environmental disruption is also sought;
- b. a fuzzy problem, that is, environmental constraints are regarded fuzzy [3];
- c. a probabilistic problem, that is, the risk of environmental disruption is minimized.

In the next sections the above models are formulated. Main control options of environmental protection are the combinations of the control of mine water withdrawals, the decrease of local transmissivity and artificial recharge.

#### Elements of the general systems model

Multiobjective control of regional industrial development and environmental protection may be described as a discrete systems model [4, 5, 6]. First, elements of this model for the general problem are presented.

#### Model elements for the general problem

Model elements are defined as follows for time  $t = 0, 1, 2, \dots$ :

- a. The input  $/I/t/$  comprises
  - i/ natural elements /wind, rainfall, evaporation, infiltration/
  - ii/ physical properties of the region /area, soil, topography, mineral resources/
  - iii/ economic, social and environmental elements /non-controllable resources such as capital, machine, manpower, requirements such as minerals, water, industrial products, migration/, environmental standards.

iv/ possible control actions, that is, decisions /controllable resources used for capacity increase, technology, environmental protection such as pollution treatment, effluent charges, artificial water recharge/. Note that elements under /i/ and /ii/ are usually stochastic.

b. The state  $S/t/$  includes the "industrial" state such as the level of industrial development or the amount of mineral resources and environmental state such as air-pollution, hydrological conditions /water levels, discharges, water-pollution/, land pollution.

c. The state transition function  $\phi$  calculates the state at time  $/t+1/$  as a function of state and input at time  $t$ :

$$S/t+1/ = \phi /S/t/, I/t// \quad /1/$$

The industrial state in  $/t+1/$  can be calculated from the existing capacity in  $t$  and inputs such as capacity increase in  $t$ .

Typical environmental state transition functions are the mass and chemical balance equations. As an example is the partial differential equation describing karstic water movement in the regional aquifer. Note that the numerical solution of such an equation is generally calculated for discrete time steps as in equation /1/.

d. The output  $R/t/$  may have elements of the state vector, especially the environmental state, say air or water pollution, karstic water level or flow. The output for the industry are the typical decision criteria, such as losses, costs, benefit, social indicators. In case of the industrial output, total-period outputs are commonly defined such as discounted costs or benefits. On the other hand, it is often not easy to interpret total-period environmental outputs since there is no ground, for instance, to summarize karstic water flows for the whole period and consider this sum as a derived environmental output. As a consequence, it is often a complicated task to find proper measures or indicators for the environmental output. You may consider separately each environment state  $R/t/$ , select the maximum, or use conceptions of fuzzy control.

e. The output function  $\psi$  calculates the output vector as function of  $S/t/$  and  $I/t/$ :

$$R/t/ = \psi [S/t/, I/t/] \quad /2/$$

The economic outputs can be calculated by the help of discounted costs and benefits. The environmental output function corresponds to the environmental state transition function.

Note that the first three elements of this model, that is  $I/t/$ ,  $S/t/$ , and  $\phi$  are sufficient for a dynamic description of the regional system considered.

Given the above model element you may select one or more outputs to be optimized and consider the rest as constraints. Now the elements of, and the systems model itself for the case study are presented.

#### Systems model for the case study

First, model elements are defined /Fig. 3/ for:

the planning horizon /years/  $t = 0, 1, \dots, T$   
 number of mines  $i = 1, \dots, M$   
 number of recharge sites  $k = 1, \dots, K$   
 number of grouting sites  $j = 1, \dots, J$   
 number of underground control points /springs, wells/  
 $n = 1, \dots, N$

For sake of simplicity, first the deterministic model is constructed:

a. The input  $I/t/$  comprises

- i/ average annual rainfall and infiltration for the region
- ii/ original rock properties for the karstic aquifer /transmissivities,  $T_r$  and storage coefficients/,  
 $R_x/i/$ : total amount of bauxite resources,  $a/i/$ : average  $Al_2O_3$  content, %,  $s/i/$ : average  $SiO_2$  content, %
- iii/ annual requirements of extracted bauxite,  $V/t/$ : quality,  $A/t/$ : min  $Al_2O_3$  /%,  $Si/t/$ : max  $SiO_2$  /% or  $M/t/$  module.
- iv/ cost functions:
  - $f_{it}$ : mining production cost
  - $fd_{it}$ : water control cost
  - $fv_{it}^k$ : artificial recharge cost
  - $fg_{jt}^k$ : grouting cost
  - $d_t$ : discount factor
- capacity limits:
  - $cx/i, t/$ : mining capacity
  - $cq/i, t/$ : mining withdrawals
  - $cv/k, t/$ : recharge
  - $cg/j, t/$ : grouting
- ideal underground flows:
  - $HJ/n, t/$ : ideal flows

v/ Possible control actions

- x/i,t/: annual bauxite extraction
- q/i,t/: annual mine water withdrawals
- v/k,t/: annual recharge amount
- tr/j,t/: relative change of transmissivity by grouting.

b. The state S/t/ includes:

the industrial state:

- X /i,t/: total amount of bauxite extracted until t
- z /i,t/: underground water level
- Tr/j,t/: total changes of relative transmissivity until t

the environmental state:

- H /n,t/: flow at the underground control points.

c. State transition functions

Industrial state:

$$X /i,t+1/ = X /i,t/ + x /i,t+1/$$

$$z /i,t+1/ = \Phi_i [z/1,t/, \dots, z/M,t/;$$

water levels

$$Tr /1,t/, \dots, Tr /J,t/; \quad q /1,t/, \dots, q/M,t/;$$

relative transmissivities                      withdrawals

$$v /1,t/, \dots, v/K,t/; \quad tr /1,t + 1/, \dots, tr/J, t + 1/]/3/$$

recharges                      relative transmissivity changes

$$Tr/j,t + 1/ = Tr/j,t/ + tr /j,t/ \quad /4/$$

Environmental state:

$$H /n, t + 1/ = \Phi_e /as in equation /3// \quad /5/$$

The state transition functions  $\Phi_i$  and  $\Phi_e$  are represented by the partial differential equation describing regional karstic water flow [7].

In fact, a finite difference numerical solution for the differential equation is considered as  $\Phi_i$  and  $\Phi_e$ .

d. The output R/t/ has industrial and environmental elements:

Industrial outputs:

1. Amount of total prod. /t/
2. Average  $Al_2O_3$  of total prod. /t/                      or average module
3. Average  $SiO_2$  of total prod. /t/                      of tot. prod.
4. Total discounted costs.

Environmental outputs:

5. Maximum deviation from ideal underground flows at each control point during the whole planning period.

e. Output functions

The above outputs can be calculated as follows:

$$1/ \sum_{i=1}^M x_{i,t}$$

$$2/ \sum_{i=1}^M a_{i/} x_{i,t}$$

$$3/ \sum_{i=1}^M s_{i/} x_{i,t}$$

$$4/ \sum_{t=0}^T d_t \left\{ \sum_{i=1}^M \underbrace{f_{it} / x_{it}}_{\text{production}} + \underbrace{f_{qit} / q_{it}}_{\text{withdrawal}} \right. +$$

$$\left. \sum_{k=1}^K \underbrace{f_{vkt} / v_{kt}}_{\text{recharge}} + \sum_{j=1}^J \underbrace{f_{gjt} / tr_{jt}}_{\text{grouting}} \right\} \quad /6/$$

$$5/ \max_{t=0, \dots, T} \{ H_{J/n,t} - H_{/n,t} \} \quad /7/$$

for  $n = 1, \dots, N$

As model elements are defined it is now straightforward to construct the model itself. The common single-objective model is reached by considering equation /6/ as objective function.

Total dis. cost  $\rightarrow$  min and the other output functions as constraints:

$$1. \sum_{i=1}^M x_{i,t} = V_{/t} \quad /8/$$

$$2. \sum_{i=1}^M a_{i/} x_{i,t} \geq A_{/t} / V_{/t} \quad /9/$$

$$3. \sum_{i=1}^M s_{i/} x_{i,t} \geq S_{/t} / V_{/t} \quad /10/$$

$$4. \quad \begin{aligned} & /HJ /n,t/ - H /n,t// \leq \text{Cons } /n,t/ \\ & \text{for } t = 0, \dots, T \\ & \quad n = 1, \dots, N \end{aligned} \quad /11/$$

The multiobjective model regards in addition to

Total disc. cost  $\rightarrow$  min.

the environmental objectives as

$$\begin{aligned} & \max /HJ /n,t/ - H /n,t// \rightarrow \text{min.} \\ & \quad t = 0, \dots, T \\ & \text{for } n = 1, \dots, N \end{aligned} \quad /12/$$

It is noted that there are three additional types of constraints:

Total extracted amount of bauxite cannot be higher than the total resources:

$$X /i,t/ \leq MX/i/ \quad /13/$$

In mines under water hazard, total extracted amount of bauxite cannot be higher than the bauxite amount above the karstic water level;

$$X /i,t/ \leq g /i, z/i,t// \quad /14/$$

where  $g$  is the amount of bauxite above  $z /i,t/$ .

Stochastic model

The quality of bauxite is known from exploration data and due to the natural uncertainty it has a random character [8]. This means that instead of the constraints /9-10/

$$E \left( \sum_{i=1}^M (\tilde{a}/i/ - m/t/ \tilde{s} /i/) x/i,t/ \right) \geq 0 \quad /15/$$

is considered, where  $\tilde{a}$ ,  $\tilde{s}$  are the average quality as random variables. In this case, there is a probability that the mining system cannot satisfy the quality constraints:

$$P_1/t/ = P \left( \sum_{i=1}^M (\tilde{a}/i/ - m/t/ \tilde{s} /i/) x /i,t/ \leq 0 \right) /16/$$

Evidently, the probability of failure should be as small as possible, that is, a new objective  $P_1/t/ \rightarrow$  min is added.

If the reliability objective is handled as a constraint

$$P_1/t/ \leq \alpha_1 \quad /t = 0, \dots, T/$$

a chance constrained multiobjective model is obtained [9, 10].

An alternative way to consider the stochastic character of bauxite quality is to involve the economic losses due to lower bauxite quality into the economic objective function /additional processing cost/. In that case, the objective function has two terms: a deterministic one  $J(\underline{x}, g)$  as specified by Eq. 6 and a stochastic one:

$$\min \left\{ J(\underline{x}, g) + \sum_{t=1}^T E \left( G \left( \sum_{i=1}^M (\tilde{a}_{i/t} - m/t/\tilde{a}_{i/t}) x_{i,t} \right) \right) \right\} / 17 /$$

where  $G$  is the loss function.

Application of the above decision models are described in several papers:

The multiobjective fuzzy model is presented in [11], a general algorithm to solve the non-linear dynamic multi-objective problem is given in [12], and the above solution techniques are applied in [13].

In the next section a numerical example shows one application.

#### Numerical example

The numerical example refers to a simplified mining and groundwater system typical for the region considered. First, data are given.

#### Data

The planning horizon is  $T=20$  years divided into four stages  $t = 1, 2, 3, 4$ .

The number of  $\left\{ \begin{array}{l} \text{mines, } M=3 \text{ including the third under water hazard} \\ \text{recharge sites, } K=1 \\ \text{grouting sites, } J=1 \\ \text{underground control points, } N=1 \end{array} \right.$

Hydrological input and aquifer properties for the region is taken from [14]. Total amount and average quality of bauxite resources are given in Table 1, while bauxite requirements, capacity limits and ideal groundwater flow can be found in Table 2. The vertical distribution of bauxite amount is illustrated in Fig. 2.

Linear costs are assumed except grouting:  $f_1 = 100$ ,  $f_2 = 320$ ,  $f_3 = 300$ , /Forint/t/,  $f_{q_1} = 880$ ,  $f_{v_1} = 1700$  / $10^3$  Ft/m<sup>3</sup>/min/stage/. Three fix grouting policies as indicated in Tables 4, 7 and 8 have been considered:

no grouting, medium grouting and heavy grouting.

The discount factor reflects 20 year lifetime and 6% discount rate.

#### State transition functions

State transition functions have been calculated according to Eq /3/. The groundwater level:

$$Z_3/t + 1/ = Z_3/t/ - 0,127/1 + T_1/ q_3 \quad /18/ \\ + 0.039 + /1 - T_1/ v_1 + 7.28$$

The groundwater control flow:

$$H/t + 1/ = 36 - 0.051/1 - T_1/ q_3 + 0,13/1 + T_1/v_1 /19/$$

Results of compromise programming [15] with objective functions /6/ and /12/ can be found in Table 3, 4 and 5. Note that results are not sensitive to changes of compromise programming weights within reasonable limits /0,4-0,6/. This is in accordance with the findings of Duckstein et al [15] for a similar regional analysis. However, there is some change if environmental protection is highly preferable /Table 6/: more recharge is needed and the underground control flow slightly increases. Comparing the three grouting policies, medium grouting seems to be the best since total discounted costs are the least and environmental control flow is acceptable. Economic optimization results in ← minimum discounted costs but the environment would be disrupted after 10 years /Table 7/.

In this numerical example, the above model choice influences the overall mining policy in a slight degree only. However, the comprehensive regional analysis has included more than one mine under water hazard; consequently, mining production policies become sensitive to the various control actions such as grouting and to model choice.

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**Table 1**  
**AMOUNT AND QUALITY OF BAUXITE**

Mine	Amount $10^3$ T	$Al_2O_3$ %	$SiO_2$
1	3000	53.7	4.9
2	7300	49.8	9.1
3	8100	52.1	5.8

**Table 2**  
**DATA FOR THE EXAMPLE**

	Stages - 4 years			
	1	2	3	4
<b>Bauxite Requirements</b>				
amount in $10^3$ T module	2500 7,2	3800 7.0	4400 6.8	4700 7.0
<b>Mining capacity limits</b>				
Mine 1	1000	1600	1600	1600
2 in $10^3$ T	900	2000	2900	2900
3	1800	2700	3200	3200
<b>Minewater withdrawal capacity limits</b>				
Mine 1 in $\text{m}^3/\text{min.}$	310	350	400	400
<b>Recharge capacity limits</b>				
in $\text{m}^3/\text{min.}$	10	30	50	50
<b>Grouting capacity limits</b>				
in relative transmissivity	0.4	0.4	0.4	0.4
<b>Ideal groundwater flow</b>				
in $\text{m}^3/\text{min.}$	30	30	30	30

Table 3

## RESULTS OF LINEAR COMPROMISE PROGRAMMING

RANGE OF WEIGHTS: 0.4-0.6

NO GROUTING

Activity	Stages in four years			
	1	2	3	4
Mine 1, Prod.	1000	1600	400	0
Mine 2, Prod.	872	1923	2377	2128
Mine 3, Prod.	623	277	1623	2572
Water withdrawal	0	242	242	388
Recharge	0	0	0	0
Underground Control Flow	36	26	26	26

TOTAL DISCOUNTED COSTS:  $3246 \times 10^6$  Forints

Table 4

WEIGHTS: 0.4-0.6

MEDIUM GROUTING

Activity	Stages in four years			
	1	2	3	4
Mine 1, prod.	1000	1600	400	0
Mine 2, prod.	900	2000	2272	2128
Mine 3, prod.	600	200	1728	2572
Water withdrawal	0	45	328	364
Recharge	0	0	42	50
Grouting	0	0.1	0.1	0
Underground Control Flow	36	31	30	30

TOTAL DISCOUNTED COSTS:  $3112 \times 10^6$  Forints

**Table 5**  
**WEIGHTS: 0.4-0.6**  
**HEAVY GROUTING**

Activity	Stages in four years			
	1	2	3	4
Mine 1, prod.	1000	1600	400	0
Mine 2, prod.	900	2000	2377	2023
Mine 3, prod.	600	200	1623	2677
Water Withdrawal	0	41	198	398
Recharge	0	0	5	34
Grouting	0	0.2	0.1	0.1
Underground Control Flow	36	34	30	30

**TOTAL DISCOUNTED COSTS: 3119 x 10<sup>6</sup> Forints**

Table 6  
 WEIGHTS: 0.3, 0.7  
 NO GROUTING

Activity	Stages in four years			
	1	2	3	4
Mine 1, prod.	1000	1600	400	0
Mine 2, prod.	900	2000	2272	2128
Mine 3, prod.	600	200	1728	2572
Water Withdrawal	0	220	295	366
Recharge	0	0	26	50
Underground Control Flow	36	27	27	27

TOTAL DISCOUNTED COSTS:  $3290 \times 10^6$  Forints

**Table 7**  
**RESULTS OF ECONOMIC OPTIMIZATION**  
**NO GROUTING**

Activity	Stages in four years			
	1	2	3	4
Mine 1, prod.	1000	1600	400	0
Mine 2, prod.	872	1975	2325	2128
Mine 3, prod.	628	225	1675	2572
Water Withdrawal	0	57	400	400
Recharge	0	0	0	0
Underground Control Flow	36	33	19	19

**TOTAL DISCOUNTED COSTS:  $3058 \times 10^6$  Forints**

List of Figures

Figure 1: The Bakony region in Hungary

Figure 2: Typical vertical distribution of bauxite resources

Figure 3: Elements of the Bakony region systems model

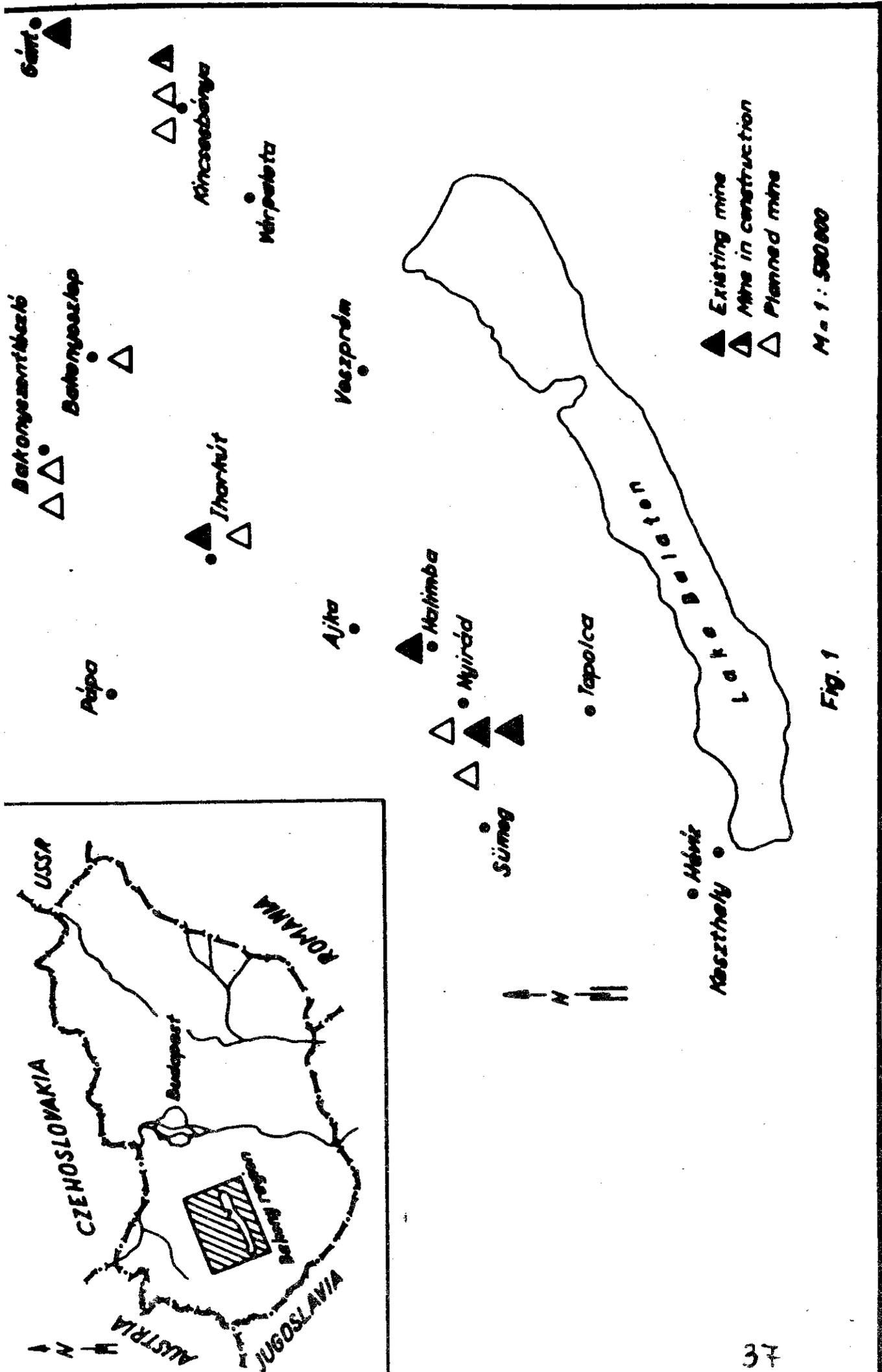


Fig. 1  
M = 1 : 500 000

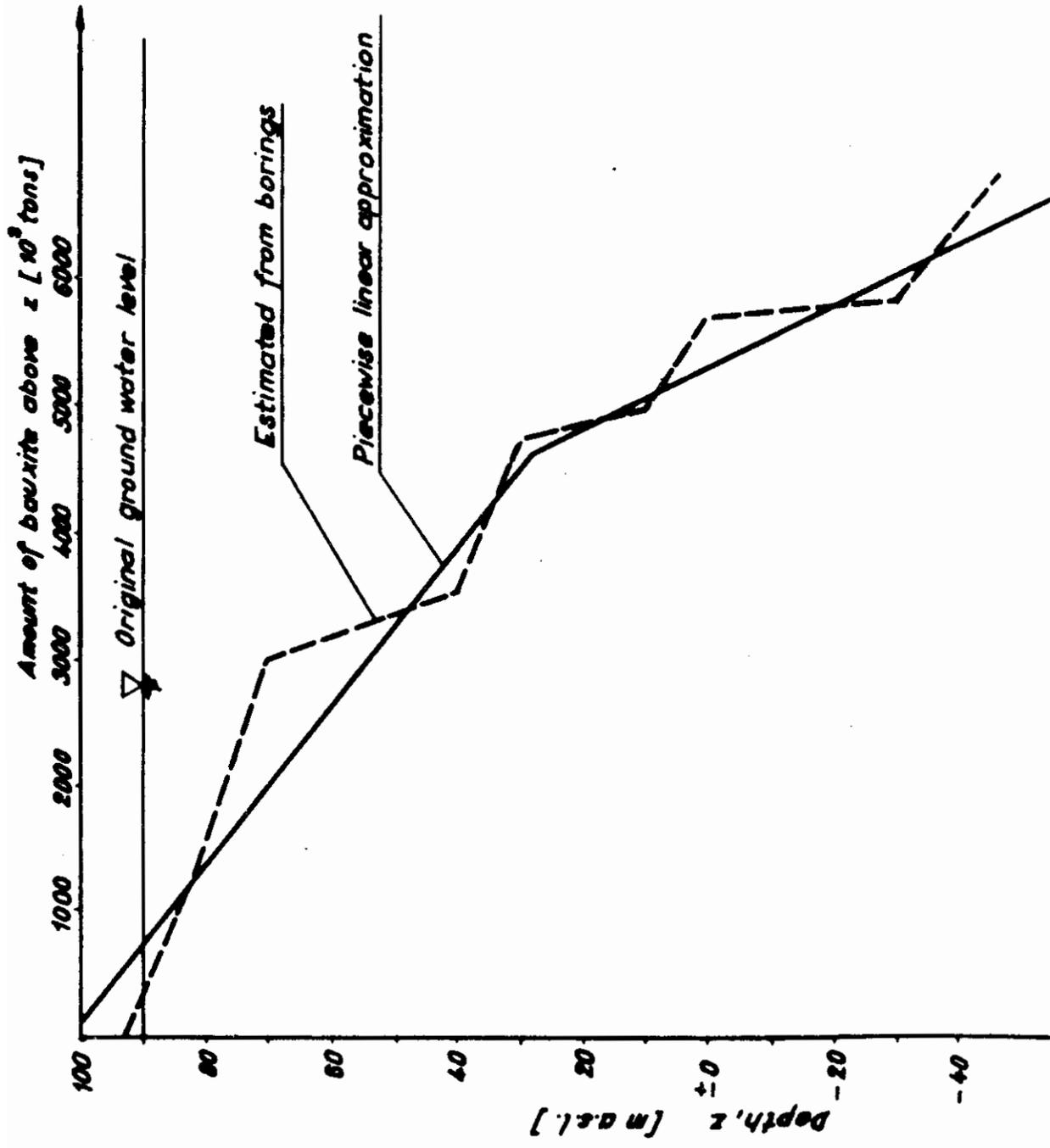


Fig. 2

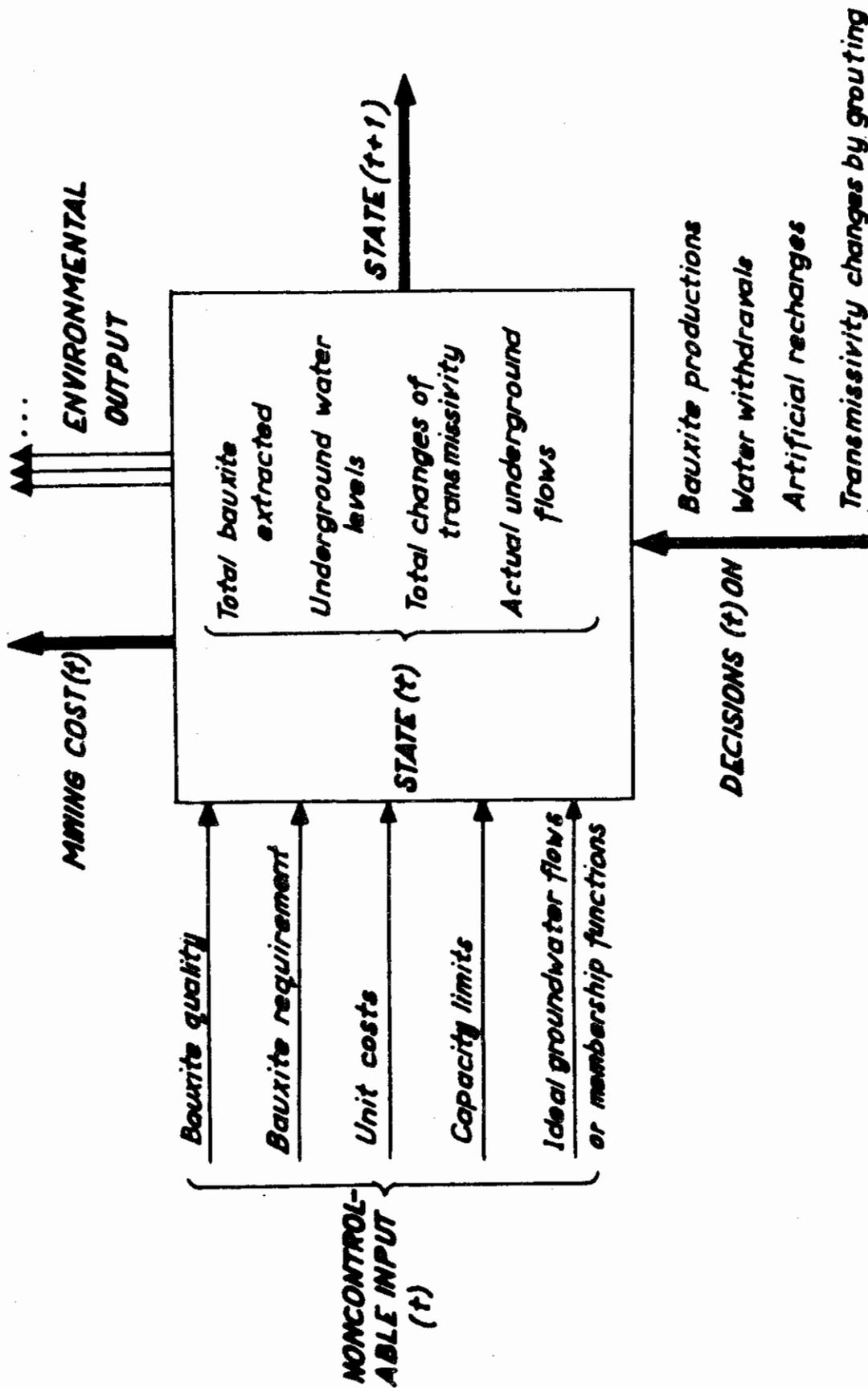


Fig. 3