

**BAYES RELIABILITY OF MINE WATER CONTROL SYSTEMS**

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**ABSTRACT**

Reliability analysis of mine water control systems is presented through an example of a combined system against karst water intrushes. The loading of the system, the yield of mine water intrushes is specified by a hydraulic model and its random parameters are determined from quasi analogous conditions. This prior information is continuously updated using local experiences by a Bayesian model. System reliability is estimated by using a failure-tree analysis and Monte Carlo simulation algorithms. A Bayesian methodology is applied to account for uncertain loading and resistance /or capacity/ statistics as well as to update reliability estimates when new system performance data are observed. Some practical consequences on the results of the reliability analysis are discussed.

**1. Introduction**

In this paper the reliability of mine water control systems is estimated using a Bayesian approach. System reliability of engineering structures is very sensitive to the accuracy

of loading and resistance statistics because the numerical value of reliability is generally found in the upper tail of the distribution, where the probability estimates are most uncertain. At the same time, the loading and resistance statistics are themselves uncertain because they are based on small samples, when not merely on regional data or experience. This motivates the use of Bayesian distributions which account for this parameter uncertainty.

The construction of underground spaces such as subways or tunnels and the operation of mines are often subject to water hazard. If an underground space is being planned or already operating under groundwater level, an inflow control system should be provided. The main elements of this system may be the drainage facilities, preliminary and subsequent grouting, the water conveyance, sediment settling and removal equipment, and the pumping stations. [5] In general, either system load or its resistance or both are random variables, so that system reliability can only be estimated statistically.

In the next section, the problem is formulated with emphasis on mining. The classical, non-Bayesian reliability model is described in the section "Non-Bayesian Reliability Model", and the Bayesian model is given in the subsequent section "Bayes Reliability Model". The failure tree method is used for constructing the model and a simulation algorithm is applied to provide the solution. A real-life example is provided in the Application section. In the last section, results of the analysis are discussed and conclusions are drawn.

## 2. Problem formulation

Reliability theory has been developed in various areas of engineering such as structural design [16], or airplane and rocket design [1]. Reliability analysis has also been applied to hydrologic problems [7], and to mining engineering where several studies, such as the analysis of the haulage, rescue, and ventilation systems can be found [10].

The first approach to the application of reliability analysis for mine water control is presented in [15]. As a result, mining regulations, standards are given in terms of an "economic reliability" for property protection and a "maximal reliability" /or a safety level/ for life protection [6]. Reliability analyses have resulted in the specification of elements such as emergency, sumps, storage spaces, rescue routes or warning systems [15].

For a rational exploitation of new coal and bauxite mines of Hungary under heavy water hazard, care should be taken to design properly the water control system, because either under-design or over-design may result in high additional

costs. For example, the cost of mine water control may reach 10-15% of the production costs under Hungarian conditions [8]

The high investment and operation costs of mine water control, and high cost of risks of flooding or disturbance of production by mine water require more advanced methods for design and operation under conditions of highly mechanized concentrated mines of large output capacity.

Along this line a non-Bayesian reliability model has been developed and tested to Hungarian mining conditions [4]. This model needs further improvement for the following reasons:

- a. statistical parameters characterizing loading and resistance conditions are uncertain during the design stage;
- b. as the construction or operation starts, observations on loading and resistance become available; this information should be utilized for an updating of the reliabilities estimated during the design stage.

It will be shown that features /a/ and /b/ can be provided by the Bayes reliability model. For illustration purposes, a combined protection system against karstic water hazard used in some Hungarian mines is now presented.

This mine water control system consists of two main subsystems:

- /1/ Protection of the production activity against the effects of mine water /called protection of faces, element 1 in Fig.1./
- /11/ Protection of blocks and the mine against flooding /called protection of mine/.

The protection of faces combines:

- the instantan drainage
- preliminary and subsequent grouting
- passive protection /water delivery from the faces/

The instantan drainage is a special control of rock-water interaction mostly in the protective layer [8] which decreases the number and the yield of spontaneous inrushes into the faces, but the total yield of mine waters /spontaneous and rained/ are considered to remain unchanged. Consequently, the use of instantan way of control protects the production activity in the faces but it has no effect on the loading of the mine water conveyance subsystem.

Grouting is intended to decrease the total yield of water. The passive way of protection involves water and sediment delivery from the faces.

The "protection of mine" subsystem corresponds to the water and sediment conveyance from every mining opening to the surface. Main elements of the analysed water delivery subsystems are roadways, tunnels /called water cuts/ for gravitationally conveying water and its sediment content from the openings into the central pumping station. The important elements are: /2/ water cut/s/ of face/s/, /3/ water cut/s/ of block/s/, /4/ the main water cut of the mine, /5/ the central plant for sediment treatment /settling and removing, /6/ the pumping station /including sumps, shaft with pumps, electric supply, etc./.

Water and sediment from block water cuts travel to the mine water cut /element 4/ which leads to the central sediment settler /element 5/ and the central pumping station /element 6/. Sediment is removed from the settler by special equipment and pumped to the surface by hydraulic means. The number of faces and blocks increases as the exploitation of the mine proceeds; the amount of inrushes and its solid particles may also increase as new underground spaces are opened.

The failure of an underground flood control system can be caused by a complex set of natural and technical factors. It is necessary to single out those failure events which are critical as far as the design and operation is concerned. More precisely, two types of events are distinguished: "disturbance of operation" and "flooding". The "disturbance of operation" corresponds to a failure event which disrupts or decreases mining production but does not stop it. The disturbance effect depends on mining technology and the number and yield of spontaneous inrushes. The effect of using the instantan drainage and grouting influences the disturbance of faces. On the other hand, "flooding" is defined as a failure which stops production altogether. Flooding occurs when the actual yield of mine water is greater than the actual capacity of the water delivery system /of faces, block, mine/. Depending on the location of the failure, the following top events are defined: /a/ disturbance of operation and flooding in faces; /b/ disturbance of operation and flooding in blocks; /c/ disturbance of operation in the mine with simultaneous disturbances in several blocks; /d/ flooding of the mine.

In the next section the classical reliability model is summarized. [4]

### 3. Non-Bayesian reliability model

Inrush events as system loading occurs as a result of inrush events which can be characterized by the following three quantities:

- I.  $q$  = magnitude of yield of inrush event;
- II.  $q_{max}$  = maximum inrush event yield into a volume of given vertical dimensions and rectangular area  $A$  of unit width;
- III.  $Q/A$  = total yield of inrush events over area  $A$ .

Pdf of these variates can be estimated as follows [4]:

#### I. Yield of inrush events

A reasonable hypothesis based on physical reasoning and strengthened by observation data is that  $q$  follows a log-normal distribution. [15]

#### II. Maximum inrush event yield over area $A$

A second hypothesis based on phenomenological reasoning and reinforced by observation data is that  $N/A$ , the number of inrush events occurring over an area  $A$ , follows a Poisson distribution with mean  $\lambda A$ . Then the asymptotic distribution of  $q_{max}$  is derived from the distributions of  $N/A$  and  $q$  as follows:

$$P/q_{max} \leq x/ = F_{q_{max}/x/} = \exp -\lambda A / 1 - F_q/x/ \quad /1/$$

It is assumed that  $q_{max}$  follows this asymptotic distribution.

#### III. Total yield of inrush events $Q/A$

The total yield for area  $A$  is calculated as the sum of a Poisson number  $N/A$  of lognormal inrushes  $q$ . [3]

$$Q/A = \sum_{i=1}^{N/A} q/i/, \quad i = 1, 2, \dots, N/A \quad /2/$$

The distribution function /DF/ of  $Q$  must be determined from the DF of  $q$  and  $N$ , since direct observation data on  $Q$  are rarely available. For this purpose, the simulation approach described in [18] appears to be appropriate. For example, this method makes it possible to account for the spatial dependence between stochastic inrush events. Note: this model has been fitted to empirical data and is practically used to predicting the mine water hazard.

Grouting activity influences the total yield of mine water. The loading of the protection system as well as the risk of disturbance and flooding depend on the control strategy.

At the same time the risk of flooding depends also on the actual performance and capacity of water delivery system. The impact of the control method on inrush yield should be considered in the reliability analysis.

A decision rule or impact function expressing the effect of control strategy on an inrush may be defined as:

$$q'_{ij} = f_{ij}/q_{ij}, a_{1ij}, a_{2ij}, a_{3ij} /3/$$

where  $q'_{ij}$  = controlled yield of intrush in face /i,j/,  
 $f_{ij}/\cdot/$  = impact function for face /i,j/,  
 $q_{ij}$  = natural yield of event and  
 $a_{1ij}, a_{2ij}, a_{3ij}$  = parameters of the impact function.

As an example, consider the control method of grouting coupled with the following decision rule:

- a/ if  $q_{ij} < a_{1ij}$ , do not grout; then  $q'_{ij} = q_{ij}$
- b/ if  $a_{1ij} < q_{ij} < a_{3ij}$ , grout a portion  $/1-a_{2ij}/$  of the intrush yield:  $q'_{ij} = a_{2ij} \cdot q_{ij}$
- c/ if  $q_{ij} > a_{3ij}$ , grout completely the intrush:  $q'_{ij} = 0$ .

To estimate the protection system reliability, events  $CC_K$  /disturbance of operation of the mine/ and  $MF$  /flooding of mine/ are considered.

Event  $CC_K$  means that there is simultaneous disturbance of operation in at least K blocks:

$$CC_K: \bigcup_{k=K}^m \left[ \bigcap_{1 \leq i_1 < i_2 < \dots < i_k \leq m} C_{i_1} \cap \dots \cap C_{i_k} / j \neq i_t \bar{C}_j \right] /4/$$

where m is the total number of blocks and  $C_i$  is the disturbance of operation in block i. Event  $C_i$  occurs when disturbance of operation is present in every face of the block. Fig. 3. shows the failure tree of event C in any of the blocks, with n being the number of faces in a block and

$$R_{ij}: /q_{max} > q_{ij}/0// /5/$$

where  $q_{ij}/0/$  is the threshold yield for face /i,j,/.

Event  $MF$  /flooding of mine/ occurs when any one of six events E, F, FR, G, H, L occurs /Fig. 4./. These events are defined as:

Event E: there is flooding in every block of the mine:

$$E: D_1 \cap D_2 \cap \dots \cap D_m /6/$$

Event F: The sediment removal capacity CH of the mine is smaller than the maximum sediment

inrush yield  $h$ . Experience shows that the following linear statistical relationship may be assumed between sediment volume  $h$  and water yield  $q_{max}$ :

$$h \sim \max_{ij} /k_0 \cdot q_{max_{ij}} + \xi / \quad /7/$$

where  $k_0$  is a specific sediment yield /tons sediment per  $m^2$  of water/ and  $\xi$  is an error term, assumed to be distributed normally with parameters  $\mu_1, \sigma_1$ . Thus, event F is:

$$F: /h > CH/ \quad /8/$$

Event FR: a failure of the mine sediment removal equipment occurs:

$$FR: /t < t/ \quad /9/$$

where  $t$  is the first failure time of the mine sediment removal equipment over time horizon  $t$ . The variate  $t$  is taken as exponential with mean  $\lambda_2$  /the expected number of failures per unit time/.

Event G: the total yield of mine inrushes,  $Q^*$  is greater than the capacity  $CQ$  of the central sediment settler /element 5/:

$$G: /Q^* > CQ/ \quad /10/$$

Event H: the total mine water yield  $Q^*$  is larger than the capacity  $CV$  of the mine water cut /element 4/:

$$H: /Q^* > CV/ \quad /11/$$

Event L: the actual capacity  $g$  of the central pumping station /element 6/ is smaller than the total yield of mine water,  $Q^*$ :

$$L: /g < Q^*/ \quad /12/$$

This event may be caused by an excessive water inrush into the mine, or failure of some of the pumps; in either case, the real capacity  $g$  of the pumping station is smaller than  $Q^*$ . The pump failure events are assumed to be exponentially distributed with parameter  $\lambda_3$ , which is the average failure rate of one pump. The number of pumps remaining in operation is a binomially distributed variate and  $g$  is the product of this binomial variate and the nominal capacity of one pump.

Since all possible failure events have been defined, the event MF of mine flooding can be written as:

$$MF: E \cup F \cup FR \cup G \cup H \cup L \quad /13/$$

Using a Monte-Carlo simulation method, failure probabilities can be estimated for several periods from a single computer run. However, if two epochs  $T_1, T_2$ , are such that  $T_1 < T_2$ , then a failure event occurring in the interval  $[0, T_1]$  also occurs in the interval  $[0, T_2]$ . Thus, simulation in  $[T_1, T_2]$  should be run conditionally on the various events in the interval  $[0, T_1]$ . The outcome of simulation runs consists in a set of sample time-series of failure probabilities, an empirical pdf of which can be constructed.

#### 4. Bayes Reliability Models

The statistical parameters in the reliability model are: the mean  $\mu$  and variance  $\sigma^2$  of inrush event yields  $q$ , the specific number  $\lambda$  of inrushes, the variance  $\sigma^2/\xi$  of sediment yield /Eq. 7/, the specific number  $\lambda_1$  of failures of a pump, and the specific number  $\lambda_2$  of failures of the sediment removal equipment.

If the values of these parameters were known, the reliability estimates would fully account for natural uncertainty. However, in the design stage, only indirect information is available, leading to uncertainty in parameter estimation. Thus reliability estimates are subject to this parameter uncertainty which decreases as operation starts and more and more observation data become available.

In the following, a Bayesian approach is used to account for both natural and parameter uncertainty [2, 13, 14]. Such a Bayesian approach has been applied to reliability engineering in [5, 11, 12]. However, its application to the reliability analysis of complex systems such as the one considered herein appears to be quite infrequent.

The present study accounts for parameter uncertainties in loading statistics. Uncertainties in  $\lambda_1$  and  $\lambda_2$  can be taken into consideration in a similar way. First, the design stage is considered, and second, the operation stage.

In the design stage, parameter uncertainty is present, and parameters are taken as random variables; a Bayesian distribution  $f(x)$  which accounts for both natural and parameter uncertainty, can be estimated as:

$$f_x(x) = \int f_x(x|\theta) f_\theta(\theta) d\theta \quad /14/$$

where  $f_x(x|\theta)$  is the model distribution given parameter vector  $\theta$ , and  $f_\theta(\theta)$  is the distribution of the parameter. The procedure below generates Bayesian values of the three loading components  $q, q_{max}, Q$  by Monte-Carlo simulation.

Magnitude of Inrush Events  $q$ .— Since  $\log q$  follows a normal distribution with /unknown/ parameters  $\mu, \sigma$ , the joint

conjugate distribution of  $\mu, \sigma$  is a normal-gamma distribution [2, 14]. This conjugate distribution has four parameters: the mean and variance of  $\mu$ , and the mean and variance of  $\sigma$ . However, only three of these parameters are independent; furthermore, any one of the parameters may be replaced by  $n$ , the number of data points /or sample size/.

Prior information on the above parameters may be determined by regional estimation, geological analogy, hydraulic calculations and/or literature data. As shown in [3, 4], the sample size  $n'$ , the mean  $m'$  of  $m$  and the variance  $s'^2/m'$  can be estimated from prior information.

The conjugate prior distribution of  $\mu, \sigma$  can be written, after [2], as

$$f'_{\mu, \sigma} / \mu, \sigma / = \left\{ \frac{1}{\sqrt{2\pi} \sigma \sqrt{n'}} \exp \left[ \frac{1}{2} \left( \frac{\mu - m'}{\sigma / \sqrt{n'}} \right)^2 \right] \right\} \times \left[ \frac{\frac{n'-1}{2} / \frac{(n'+1)}{2}}{\Gamma \left( \frac{n'+1}{2} \right)} \left( \frac{s'^2}{\sigma^2} \right)^{(n'-1)/2} \exp \left( -\frac{n'-1}{2} \frac{s'^2}{\sigma^2} \right) \right] \quad /15/$$

The Bayesian simulation algorithm is as follows:

1. Generate a pair  $\mu, \sigma$  using the normal-gamma distribution /Eq. 15/ with prior  $m'$ ,  $s'$  and  $n'$ .
2. Generate a realization of a normal variate  $n_j$  using parameters  $\mu, \sigma$  generated in step 1.
3. A prior Bayesian random value of  $q$  is  $q_j = \exp /n_j/$ .

Maximum Inrush Event Yield over Area  $A = q_{max}$ .— The additional uncertainty caused by parameter  $\lambda$  /Eq. 1/ is entered into the estimation of the DF of  $q_{max}$ .

The conjugate distribution of a  $\lambda$  is a gamma-2 distribution with two prior parameters  $\alpha'$  and  $\beta'$  such that:

$$m' / \lambda / = \frac{\alpha'}{\beta'} \quad \text{and} \quad s'^2 / \lambda / = \frac{\alpha'}{\beta'^2} \quad /16/$$

$m' / \lambda /$  and  $s'^2 / \lambda /$  are estimated from available prior information [4]. The same type of gamma-2 distribution defines the conjugate distribution of  $\lambda_2$  and  $\lambda_3$ . A Bayesian distributed value  $N_j$  can be simulated as follows:

1. Generate a value of  $\lambda$  using a gamma-2 distribution with prior parameters  $\alpha'$  and  $\beta'$  /Eq. 16/.

2. Generate a /prior/ Bayesian Poisson variate  $N_j$  using parameter  $\lambda$  found in step 1.

The simulation procedure for estimating a Bayesian distribution of  $q_{max}$  is:

1. Generate a random value of  $N_j$ .
2. Generate a number  $N$  of /prior or posterior/ realization of  $q$ :  $q_1, q_2, \dots, q_N$ .
3. Find the value  $q_{max} = \max_{1 \leq i \leq N} q_i$ ; iterate and obtain an empirical distribution of  $q_{max}$ .

Total Yield  $Q$  of Inrush Events Over Area  $A$ .-- Using Equation /2/, a Bayesian distribution of  $Q$  can be simulated in the following way:

1. Generate a Bayesian distributed value of  $N_j$ .
2. Generate a number  $N$  of Bayesian distributed values of  $q$ :  $q_1, q_2, \dots, q_N$ .
3. A Bayesian sample value of  $Q$  is:  $Q = \sum_{i=1}^N q_i$ .

Next, in the operation stage, assume that a number  $n$  of inrush events has been observed over an area  $A_0$  up to period  $t < T$ : let  $m$  be the mean of the logarithm of observed inrush yields, and  $s^2$  the variance of the same quantity. Bayesian distributions are used to obtain more accurate reliability estimates for the subsequent periods  $t > T$ . Since prior distributions belong to a conjugate family, the posterior /updated/ parameter distribution is of the same type as the prior, and the parameters can readily be calculated as before. Updated Bayesian variates can then be generated by the algorithm given in the design stage.

## 5. Application

The reliability model has been tested and experimentally applied using quasi realistic data of a mine being planned in the Transdanubian region at a depth of 300 metres below the karstic water level.

The system and elements of mine water control are presented in section 2.

Parameters of the impact function /Eq. 3/ are given in Table 1. The same decision rule is used for every face of the mine in every period. Input data for the reliability estimation are given in Table 2. Note that these data available or can be determined [15, 3] in mines or tunnels subject to water hazard.

The non-Bayesian reliability model utilizes the loading statistics given in Table 2. Table 3 gives prior information on loading and resistance statistics for the Bayes model. Tables 4 and 5 illustrate results of applications of the non-Bayesian and the Bayesian models, respectively. Prior information on  $\lambda_2$  and  $\lambda_3$  is inferred from performance data of, respectively, submerged pumps and sediment removal equipment.

The effect of different grouting strategies is reflected in the numerical values of failure probabilities for both non-Bayesian and Bayesian models. On the other hand, the Bayesian reliability estimates are smaller than the non-Bayesian ones, because, as it can be expected, parameter uncertainty increases the estimates of failure probabilities.

In the next step, it is assumed that a mine with input data given in Table 2 and prior loading and resistance given in Table 3 has been operating for 6 years. During this time,  $n = 78$  inrush events have been observed over the total area of faces,  $A_d = 2,6 \text{ km}^2$ . The mean and standard deviation of the logarithm of observed inrush yields are:  $m = 0.4$ ,  $s = 0,5$ . Table 6 shows the updated Bayesian reliabilities estimated for the subsequent time periods /4,5,6/, without any further change in the original input data. It can be seen that expected failure probabilities have become smaller, because of the smaller number and magnitude of observed inrushes than predicted in prior information /Table 3/. At the same time, the failure probabilities are sensitive to the different grouting strategies.

#### DISCUSSION AND CONCLUSIONS

A Bayesian reliability system model has been applied to a quasi real-life mine water control system. In contrast with a classical reliability model, this Bayesian approach accounts for the uncertainty in statistical information on loading and/or capacity. The fact that Tables 5 and 6 exhibit fairly high values of tested system failure probabilities prompts the following two remarks:

a./ One of the critical events leading to mine flooding is the failure of the sediment removal equipment. As numerical results show, for a specific failure time  $\lambda_2 = 0,02/\text{year}$ , it is quite probable to have such a failure<sup>2</sup> event within the lifetime of the mine. As a redundancy, provisional storage of removed sediment is highly necessary. As an outcome of this analysis, improved underground sediment settling plan will be equipped by sediment storage facilities.

b./ The other weak point in the system is the failure event of submerged pumps. In the numerical example, no stand-by was considered, while in reality, at least 2 stand-by pumps

are available for the given mining capacity. Using standard reliability theory [16], it is straightforward to include a given number of standbys into the reliability model.

Based on the results presented in Tables 5 and 6 practical measures can be proposed as follows:

- Since failure rate of pumps may be higher in the last part of their life time, a programmed maintenance is necessary for increasing the performance of the pumping station. This solution seems to be more economic than the increase of the number of standby units.

- In case of large parameter uncertainty an increase of the number of standby units cannot be disregarded though the risk of an extremely great water inrush may be small. In these cases, provisional modes for increasing the standby pumping capacity seem to be more economic. This principle is used in almost every new ecene mine under construction.

The law of large numbers permits to determine the relationship between the error bound  $\xi_s$  of simulation and the number  $n$  of samples simulated [7]:

$$\xi_s = \frac{1}{\sqrt{4n/1-p_0}} \quad /17/$$

where  $p_0$  is a given probability level. In this application  $n = 1000$  events were simulated in each case; with a probability of  $p_0 = 90\%$ , Eq. /17/ yields an error bound of  $\xi_s = 0,05$ .

The results presented in this paper point to the following conclusions:

1. The reliability estimation of an underground engineering system, such as a minewater control system, should account for natural uncertainty in both loadings and resistances.
2. The estimation procedure can be displaced as a failure tree.
3. A spatial event-based stochastic model can be used to characterize water inflow quantities, that is, the loading of the mine water control system.
4. Resistance of the system corresponds to the capacity of control elements.
5. System reliabilities for both the non-Bayesian and Bayesian models can be estimated by the simulation techniques developed herein.

6. The effect of loading control by use of rock grouting can be accounted for by an impact function.

7. Bayesian reliability estimation is to be used in cases when /a/ uncertain loading and/or capacity statistics are available in the design stage, and /b/ earlier operational experience is available and more accurate reliability estimates are sought for subsequent operation periods.

8. Results of the numerical example on a combined mine water control system have given valuable information on how to increase system reliability in an efficient way. There is an urgent need to apply this methodology for practical design since both mining investment costs and economic losses due to systems failure are high.

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Table 1.  
Parameters of the Impact Function

Parameters $\frac{r^2}{\text{min}}$	Alternatives		
	Passive Control or Instantan- Drainage	Moderate Grouting	Intensive Grouting
	A	B	C
$a_1$	30	0,35	0,2
$a_2$	1	0.5	0.3
$a_3$	100	9,0	1,3

Table 2.  
Data for the Estimation of System Reliability

Time period	Number of blocks	Number of faces in a block	Area of a face A km <sup>2</sup>	Specific number of in-rushes 1/km <sup>2</sup>	Data on inrush events	
					Average yield m <sup>3</sup> /min	Statistics of the log-normal distribution m s
1	1	1	0.10	40	1.21	-0.3 0.48
2	2	2	0.15	40	1.2	-0.3 0.48
3	4	2	0.25	40	1.2	-0.3 0.48
4	5	2	0.6	60	2.0	0 0.48
5	5	2	0.6	60	2.0	0 0.48
6	5	1	0.2	40	2.0	0 0.48

Table 2.-- continued

- $q_{ij}/O/$  = inrush yield threshold for disturbance of operation
- $cf_{ij}$  = water conveyance capacity from faces
- $Cv_i$  = capacity of block water cuts
- $k_0$  = specific yield of sediment
- $S^2/t/$  = variance of sediment estimation
- CH = sediment removal capacity
- CQ = water capacity of the control sediment settler
- CV = water conveyance capacity of the mine water cut
- n = number of pumps
- $\lambda_2$  = specific number of failures of a pump
- $\lambda_3$  = specific number of failures of the sediment removal equipment

Table 3.  
 Prior Information on Loading and Resistance

Loading statistics

Time periods	m'	s'	n'	m'/ $\lambda$	s'/ $\lambda$
1	-0.3	0.4	9	4	2
2	-0.3	0.4	9	6	3
3	-0.3	0.4	9	10	5
4	0	0.4	9	36	18
5	0	0.4	9	36	18
6	0	0.4	9	8	4

Resistance statistics:

For all six time periods

$$m'/\lambda_2 = 0.11, s'/\lambda_2 = 0.05, m'/\lambda_3 = 0.02, s'/\lambda_3 = 0.007$$

Table 4.

Non-Bayesian Failure Probabilities for the Whole Mine

Probability of Disturbance of operation	Time periods					
	1	2	3	4	5	6
A. Passive control & instantan- drainage	0.0	0.0	0.0	0.0	0.0	0.0
B. Moderate grou- ting	0.0	0.0	0.0	0.0	0.0	0.0
C. Intensive grou- ting	0.0	0.0	0.0	0.0	0.0	0.0
<hr/>						
Probability of Flooding	/for the first variant/					
A.	0.065	0.137	0.466	1.0	1.0	1.0
B.	0.060	0.136	0.299	0.981	1.0	1.0
C.	0.060	0.136	0.299	0.569	0.981	1.0

Table 5.

Bayesian Failure Probabilities for the Whole Mine

Probability of disturbance of operation	Time periods					
	1	2	3	4	5	6
A	0.006	0.006	0.006	0.006	0.006	0.059
B	0.0	0.0	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	0.0	0.0	0.0

  

Probability of flooding /for the first variant/						
A	0.086	0.167	0.683	1.0	1.0	1.0
B	0.080	0.153	0.462	0.990	1.0	1.0
C	0.072	0.145	0.361	0.662	0.995	1.0

Table 6.  
 Results of the Updated Bayesian Failure Probabilities  
 /for the first variant/

Probability of disturbance of operation	Time periods		
	4	5	6
A	0.0	0.0	0.0
B	0.0	0.0	0.0
C	0.0	0.0	0.0
<hr/>			
Probability of flooding			
A	0.380	1.0	1.0
B	0.360	1.0	1.0
C	0.360	0.594	0.824

Appendix I. - Notation

- A = surface area of underground space
- $a_1, a_2, a_3$  = parameters of the impact function
- $CC_K$  = event of simultaneous disturbance of operation at least in a number K of blocks
- $C_i$  = event of disturbance of operation in block i
- $D_i$  = event of flooding in block i
- E = event of flooding in every block
- F = probability density function
- FC = event of insufficient sediment removal capacity
- FR = event of failure of the sediment removal equipment
- G = event of insufficient sediment settler capacity
- H = event of insufficient mine water cut capacity
- h = volume of sediment
- i = serial number of blocks i: 1, ..., m
- j = serial number of faces in a block j: 1, ..., n
- L = event of insufficient pumping station capacity
- m = empirical mean
- $m'$  = prior information
- $m''$  = posterior information
- $\bar{m}$  = expectation of log q distribution
- MF = event of mine flooding
- $N/A$  = number of inflow events over area A
- n = the number of data points
- $Q/A$  = total yield of inflow over area A
- q = inflow event yield

$q_{max}$  = maximum inflow event yield over area A  
 $R_{ij}$  = event of disturbance of operation in face /i,j/  
 $s^2$  = empirical variance  
 $t$  = stages of operation  $t: 1, \dots, T$   
 $K$  = parameter of the conjugate gamma distribution  
 $K$  = real capacity of the pumping station  
 $\lambda$  = parameter of the conjugate gamma distribution  
 $\sigma$  = standard deviation  
 $\epsilon$  = error term in sediment estimation  
 $N$  = a normal variate  
 $\underline{\mu}$  = vector of statistical parameters  
 $\lambda$  = specific number of inflow events  
 $:$  = random variable