NON-LINEAR FLOW OF GROUND WATER DURING MINE DEWATERING OPERATIONS

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ABSTRACT

Darcy's law has been currently utilized for describing flow of groundwater, but it is not suitable in certain cases in which high hydraulic conductivities and/or big hydraulic gradients are involved, that is why it has been necessary to look for nonlinear laws to describe the flow. The nonlinear laws proposed by different investigators have not been satisfactorily related with the hydraulic properties of the media, but the author has found expressions with coefficients that are function of intrinsic hydraulic properties and has established the basis for giving a new general nonlinear approach to groundwater flow. Some of the research results obtained by applying the new nonlinear approach that can be useful in the sizing and construction of water table drawdown systems in mines are reported, such as: the new criteria for the definition of the existing regime of flow; the general picture of the flow towards a pumping well and the determination of characteristic radial distances in the flow field around it; the new equations for analyzing steady and unsteady nonlinear flow towards wells and the new equations for describing steady nonlinear flow towards galleries and trenches.

INTRODUCTION

Darcy's law has been currently utilized for describing flow of groundwater. This law, that established a linear relationship between velocity and hydraulic gradient is not suitable in certain cases in which high hydraulic conductivities and/or big hydraulic gradients are involved that is why it has been necessary to look for nonlinear laws to describe the flow. Until recently the equations obtained by different investigators have not been satisfactorily related with the hydraulic properties of the media. As result of his research work it has been possible for the author to find expressions for the nonlinear law of flow in which the coefficients are function of intrinsic hydraulic properties of the media, and to establish the basis for giving a new general nonlinear approach to groundwater flow.

The purpose of this paper is to report some of the new results obtained with the application of the nonlinear approach to groundwater flow, such as: the new criteria for the definition of the existing regime of flow, the general picture of the flow towards a pumping well and the determination of characteristic radial distances in the flow field around it; and the new equations for describing nonlinear flow towards wells, galleries and trenches obtained in terms of characteristic hydraulic properties of the groundwater reservoir. As nonlinear flow can occur during mine deatering operations, the research results reported herein can be utilized in the sizing and construction of water table drawdown systems.

NON-LINEAR LAWS OF GROUNDWATER FLOW

Taking into account the observed deviations of the Darcy's law a great number of experiments have been performed to determine its range of validity, and as a consequence, several equations have been proposed for the law of non-Darcian (nonlinear) flow through porous media. In general these equations can be classified in two types: monomial and binomial [1], that can be expressed respectively as follows:

$$U = K_n I^n \qquad (1)$$

$$I = aU + bU^2 \qquad (2)$$

where:
- U, apparent velocity of seepage
- K_n, hydraulic conductivity of the medium for the regime of flow with exponent n
- n, exponent of the law of flow (varies between 1 and 0,5)
- I, hydraulic gradient
- a, constant with dimensions
- b, constant with dimensions

Until recently the coefficients of both types of equations were not satisfactorily related with the hydraulic properties of the media. Upon the basis of the research results obtained by him and several investigators, especially those of Ward [2], the author [3] has found the way to express the nonlinear law of flow through porous media in binomial or monomial form as function of intrinsic hydraulic properties of the media. These properties are: the intrinsic permeability, k, already known; and a new propertie named equivalent roughness of the media, C, [3,4] In this way the monomial expression keeps the form of equation 1, but being possible now to express K_n as a function of k and C; and the binomial expression of the law takes the form:

$$I = \frac{\nu}{gk} U + \frac{C}{gk^{1/2}} U^2 \qquad (3)$$

where: ν, kinematic viscosity of the fluid
g, acceleration of gravity

The new monomial form of the law as well as the binomial include as a particular case the Darcy's law ($U=K_D I$).

As it is known, in practice, it has been customary to utilize instead of k, the Darcian hydraulic conductivity, K_D, and the Darcian transmissibility, $T_D = m K_D$ (where m=saturated thickness of the aquifer), as characteristic hydraulic properties of the media, although they are dependent on the fluid viscosity.

By analogy with the Darcian conductivity and transmissibility, it has been defined [5] the turbulent hydraulic conductivity, K_T, and the turbulent transmissibility, T_T; establishing besides the following functional relationships of the different conductivities with the intrinsic properties of the media.

$$K_n = \left[\frac{gk^{1/2}}{\frac{1}{N_{RK}} + C} \right]^{1/2} I^{1/2 - n} \qquad (4)$$

$$K_D = \frac{gk}{\nu} \qquad (5)$$

$$K_T = \left[\frac{gk^{1/2}}{C} \right]^{1/2} \qquad (6)$$

where: N_{RK}, Ward's Reynolds number ($N_{RK} = U k^{1/2}/\nu$)

By combining equations (3),(5) and (6) the binomial law of flow can be expressed as:

$$I = \frac{U}{K_D} + \frac{U^2}{K_T^2} \qquad (7)$$

On the basis of the new expressions obtained for the nonlinear law of flow in monomial (equation 1 combined with eq. 4) or binomial form (equations 3 and 7), it has been possible to devise a new theoretical basis with nonlinear approach for groundwater hydraulics. This more general approach makes it possible to analize besides as particular cases the situation in which Darcian flow occurs.
The concepts established above are fully applicable to loose clastic sediments and to fractured rocks with fine

structure, and although in the case of other fractured rocks the continuum approach cannot be applied without restrictions, there are many cases in which the results of field tests in cuban carbonate rock aquifers conform fairly well with the analytical expressions derived from the binomial law of flow [5] as represented by equations 5 and 7.

NEW CRITERIA FOR THE DEFINITION OF THE EXISTING REGIME OF FLOW IN SATURATED MEDIA.

The proposed binomial equations are best suited than the monomial for describing the flow within ample range of gradients, and that is why can be considered as "exact" general laws of flow. These equations can be reduced to monomial form in two particular situations [6]. When N_{RK} is little enough, the flow will be linear and the second term of the second member of the equations may be canceled, obtaining the approximate expressions:

$$I = \frac{\nu}{gk} U = \frac{U}{K_D} \qquad (8)$$

in which we can recognize Darcy's law.

When N_{RK} is large enough the flow will be turbulent, and the first term of the second member of the equations may be cancelled, obtaining the approximate expressions:

$$I = \frac{C}{gk^{\frac{1}{2}}} U^2 = \frac{U^2}{K_T^2} \qquad (9)$$

in which we can recognize the pure turbulent law of flow as proopposed by the author before [7].

If we accept an error, E, in the application of the approximate laws represented by equations 8 and 9 as compared with the binomial exact nonlinear law, it is possible to establish the following critical Reynolds numbers [6]:

$$N_{RKD} = \frac{E}{C} \qquad (10)$$

$$N_{RKT} = \frac{1-E}{CE} \qquad (11)$$

where: N_{RKD}, Ward's Reynolds number for the higher limit of application of Darcy's law.
N_{RKT}, Ward's Reynolds number for the lower limit of application of the pure turbulent law of flow.

In field practice it is not easy to determine the places where these values of N_{RK} occur, nevertheless it is easy to measure hydraulic gradients, that is why we have derived expressions that for a practical value of E=0,05 give:

$$I_{crd} = \frac{0,05}{gCk^{3/2}}^{2} = 0,05\left(\frac{T_T}{T_D}\right)^2 \qquad (12)$$

$$I_{crt} = \frac{361}{g\,Ck^{3/2}}^{2} = 361\left(\frac{T_T}{T_D}\right)^2 \qquad (13)$$

where: I_{crd}, critical gradient corresponding to N_{RKD}
I_{crt}, critical gradient corresponding to N_{RKT}

The existing regime of flow will be easily determined by comparison of the existing gradient with the critical values obtained from equations 12 and 13

GENERAL PICTURE OF THE FLOW AROUND A PUMPING WELL

Applying the concepts of critical gradients and Reynolds numbers referred above to the flow around a pumping well, it has been possible to define the Darcian radius, r_D, and the turbulent radius, r_T, as function of the discharge of the well, Q, and the hydraulic properties of the aquifer [8]. These radii for E=0,05 are represented as:

$$r_D = \frac{Ck^{\frac{1}{2}}}{0,05\,\nu} \cdot \frac{Q}{2\pi m} = \frac{Q}{0,1\pi} \cdot \frac{T_D}{T_T^2} \qquad (14)$$

$$r_T = \frac{Ck^{\frac{1}{2}}}{19\,\nu} \cdot \frac{Q}{2\pi m} = \frac{Q}{38\pi} \cdot \frac{T_D}{T_T^2} \qquad (15)$$

Consequently, for E=0,05 it results:

$$r_D = 380\, r_T \qquad (16)$$

By a simple comparison of r_D and r_T with the well radius, r_o, it will be possible to determine the different regimes of flow existing around the well and the corresponding zones limited by the values obtained for r_D and r_T for each Q. In this way a comprehensive picture of the flow around the well will be obtained, determining also if it is adequate to utilize linear (Darcian) concepts for analyzing the flow or it is necessary to use the nonlinear approach.

NON-LINEAR FLOW TOWARDS A PUMPING WELL

Until recently the analysis of nonlinear flow towards wells has been done utilising almost exclusively the monomial exponential form of the law of flow [1]. The results obtained were greatly limited in their application, because K_n can only be considered as a constant within narrow limits [3]. In the other hand some investigators [9,10,11,12] utilizing binomial laws of flow have obtained equations for nonlinear steady flow towards wells but they failed to identify the coefficients of their equations with characteristic hydraulic properties of the media.

As result of the research done by the author [3,15], equations with a binomial approach have been obtained for steady and unsteady confined nonlinear flow towards wells as function of the intrinsic hydraulic properties of the media. The general conditions considered are the customary: isotropic and homogeneous aquifer, well penetrating its full thickness, etc.

For unsteady conditions the equation is expressed by:

$$S_r = \frac{\nu Q}{2\pi m g k} W(u) + \frac{C}{g k^{\frac{1}{2}}} \cdot \frac{Q^2}{4\pi^2 m^2} \cdot \frac{r_o - r}{r \cdot r_o} \quad (17a)$$

and also as: $$S_r = \frac{Q}{4\pi T_D} W(u) + \frac{Q^2 (r_o - r)}{4\pi^2 T_T^2 \, r \, r_o} \quad (17b)$$

where: S_r, drawdown at a radial distance, r, from the center of the pumping well
$W(u)$, Theis' well function
r_o, radius of influence

A close examination of equation 17b permits to identify the first term of the second member of the equation as the equation of Theis [14] for the drawdown for unsteady linear flow, and the second term as the drawdown for pure turbulent flow [7] meaning that the total drawdown is made up of two components, enlighting in this way the real behaviour of the process. For times large enough (more than 60 minutes in the case of Cuba) Jacob's approximation can be utilized in lieu of Theis equation and equations 17 will be expressed as [3,15].

$$S_r = \frac{Q}{4\pi T_D} \ln \frac{2{,}246\, T_D t}{r^2 E} + \frac{Q^2 (r_o - r)}{4\pi^2 T_T^2 \, r \, r_o} \quad (18)$$

where: t, time since pumping started
E, storage coefficient

Aquifer's properties can be determined by simple analytical procedures applying equation 18 to data resulting from pumping tests, being unnecessary to utilize the cumbersome processes of analysis and of type curve fitting traditionally used with unsteady well flow formulae. [3,7].

For steady conditions the linear component of the drawdown is reduced to the Thiem form and consequently:

$$S_r = \frac{Q}{2\pi T_D} \ln \frac{r_o}{r} + \frac{Q^2 (r_o - r)}{4\pi^2 T_T^2 \, r \, r_o} \quad (19)$$

The binomial nonlinear approach has also been applied to step-drawdown tests [15].

Although equations 17, 18 and 19 have been deduced for a confined aquifer they can also be utilized in unconfined aquifers of relatively large saturated thickness.

An important consequence is also derived from appling equations 18 and 19 to the drawdown at the pumping well. It is, that contrary to the accepted assumption that the variationes of diameter of the well do not affect the discharge or the drawdown; when nonlinear flow occurs it is evident that those variations of the diameter, influence both; drawdown and discharge [1,17].

STEADY NON-LINEAR FLOW TOWARDS GALLERIES AND TRENCHES

The flow towards galleries and trenches occurs in general without great variations of the hydraulic gradient along the direction of flow, that is why the exponential monomial approach as well as the binomial can both be utilized in the analysis of nonlinear flow towards these water abstracting structures.

We will consider here two general cases:

a) Galleries and trenches penetrating the full thickness of the aquifer
b) Trenches penetrating only the upper part of a deep aquifer

The analysis will be done for steady flow utilizing exponential and binomial approach, in confined and unconfined homogeneous and isotropic aquifers.

GALLERIES AND TRENCHES PENETRATING THE FULL THICKNESS OF A CONFINED AQUIFER

In this case schematically represented in figure 1 the flow towards one of the sides of the structure, can be expressed as:

$$q = UA = Um \quad (20)$$

$$q_t = 2q \quad (21)$$

where: q, discharge towards one side of the gallery or trench per unit length
A, flow area per unit length of structure
q_t, total discharge through both sides

If we introduce into equation 20 the corresponding value of U in accordance with the law of flow considered we will obtain the pertinent equations for calculating q.

Analysis with the exponential law of flow.

By referring to figure 1, it can be seen that: [1,16]

$$I = \frac{dh}{dx} \qquad (22)$$

where: h, piezometric level of ground water at the distance x
x, distance measured from the face of the gallery or trench

By conveniently combining equations 1, 20 and 22 we can obtain the differential equation:

$$q^{\frac{1}{n}}\, dx = m^{\frac{1}{n}} K_m^{\frac{1}{n}}\, dh \qquad (23)$$

Integrating equation 23 within the limits h=h, $h=h_g$; x=0; x=x, we obtain:

$$q = m\, K_n \left(\frac{h-h_g}{x}\right)^n \qquad (24)$$

For the particular case of linear flow, $K_n = K_D$ and n=1, transforming equation 24 into:

$$q = m\, K_D \left(\frac{h-h_g}{x}\right) \qquad (25)$$

In the case of pure turbulent flow, $K_n = K_T$ and n=0,05, resulting in:

$$q = m\, K_T \left(\frac{h-h_g}{x}\right)^{\frac{1}{2}} \qquad (26)$$

Analysis with the binomial law of flow.

By conveniently combining equations 7, 20 and 22 the following differential equation is obtained: [16]

$$\frac{dh}{dx} = \frac{q}{m\, K_D} + \frac{q^2}{m^2 K_T^2} \qquad (27)$$

Integrating equation 23 within the limits $h=h_o$, $h=h_g$, x=0 and $x=x_o$ we obtain:

$$h_o - h_g = S_g = \left(\frac{q}{m\, K_D} + \frac{q^2}{m^2 K_T^2}\right) x_o \qquad (28)$$

where: S_g, drawdown at the face of the gallery or trench

Equation 28 is of general application to any type of flow (linear or nonlinear).

From equation 28 we can obtain:

$$q = \frac{-\frac{1}{m K_D} + \sqrt{\frac{1}{m^2 K_D^2} + \frac{4 x_0 S_g}{m^2 K_T^2}}}{\frac{2}{m^2 K_T^2}} \qquad (29)$$

GALLERIES AND TRENCHES PENETRATING THE FULL SATURATED THICKNESS OF AN UNCONFINED AQUIFER

In this case schematically represented in figure 2, the flow per unit length towards one side of the structure through a saturated cross-section of height h, is expressed as:

$$q = UA = Uh \qquad (30)$$

Utilizing Dupuit assumptions and conveniently combining equations 1, 22 and 30 we can obtain the differential

$$q^{\frac{1}{n}}\, dx = K_n^{\frac{1}{n}}\, h^{\frac{1}{n}}\, dh \qquad (31)$$

Integrating equation 31 within the limits h=h, h=h_g; x=0 and x=x we obtain [1,16]

$$q = K_n \left[\frac{h^{1+\frac{1}{n}} - h_g^{1+\frac{1}{n}}}{\left(1 + \frac{1}{n}\right) x} \right]^n \qquad (32)$$

For the particular case of linear (Darcian) flow, equation 24 transforms into:

$$q = \frac{K_D}{2x}(h^2 - h_g^2) \qquad (33)$$

In the case of pure turbulent flow, it results:

$$q = K_T \left[\frac{h^3 - h_g^3}{3\, x} \right]^{\frac{1}{2}} \qquad (34)$$

TRENCHES PENETRATING ONLY THE UPPER PART OF A DEEP AQUIFER

In this case represented schematically in figure 3, the flow pattern permits to consider as if it were through a semi-cylindrical surface. Then the area of flow per unit length will be:

$$A = \frac{2\pi x}{2} = \pi x \qquad (35)$$

and the total discharge per unit length:

$$q_t = UA = x \pi U \qquad (36)$$

The analysis can be made with exponential or binomial approach.

Exponential approach

Taking into account that $I=dy/dx$, and conveniently combining equations 1 and 36 it results that [16]

$$q_t^{\frac{1}{n}}\, x^{-\frac{1}{n}}\, dx = (\pi K_n)^{\frac{1}{n}}\, dy \qquad (37)$$

Integrating equation 37 within the limits $x=x_1$, $x=x_2$, $y=y_1$ and $y=y_2$ we obtain

$$q_t = \pi K_n \left[\left(1-\frac{1}{n}\right) \frac{y_2-y_1}{x_2^{\frac{n-1}{n}} - x_1^{\frac{n-1}{n}}} \right]^n \qquad (38)$$

Equation 32 can be utilized for any type of nonlinear flow ($1 < n \leqslant 0{,}05$). For $n=0{,}5$ (pure turbulent flow) it results that:

$$q_t = \pi K_T \left[\frac{x_1 x_2\,(y_2-y_1)}{x_2-x_1} \right]^{\frac{1}{2}} \qquad (39)$$

A close examination of equation 38 reveals that it cannot be utilized in the case of linear flow ($n=1$). It can be demonstrated [16] that in that case:

$$q_t = \frac{\pi K_D\,(y_2-y_1)}{\ln\,(x_2/x_1)} \qquad (40)$$

At the trench, x is assumed to be equal to the radius of the half-cylindrical trench of the same flow-entrance area; that is:

$$x_1 = r = (W+2a)/\pi \qquad (41)$$

Binomial approach

Taking into account that $I=dy/dx$, and conveniently combining equations 7 and 36, the following differential equation results:

$$\frac{dy}{dx} = \frac{q_t}{K_D\,\pi\, x} + \frac{q_t^2}{K_T^2\,\pi^2 x^2} \qquad (42)$$

Integrating equation 42 within the limits $y=y_1$, $y=y_2$, $x=x_1$ and $x=x_2$, we obtain:

$$y_2-y_1 = \frac{q_t}{\pi K_D} \ln \frac{x_2}{x_1} + \frac{q_t^2\,(x_2-x_1)}{\pi^2 K_T^2\, x_1 x_2} \qquad (43)$$

Equation 43 is general and applicable to any regime of flow. A close examination of it allows to recognize a Darcian component and a turbulent (quadratic) component. The value of q_t can be easily deduced from this equation.

In practice we can assume that the quadratic component is nill for Darcian flow and then equation 40 will be obtained from equation 43. In a similar fashion it can be supposed that the linear component is of no account for pure turbulent flow and equation 39 will be obtained from equation 43.

CONCLUSIONS

Due to the high hydraulic gradients and/or large hydraulic conductivities involved nonlinear flow can occur during mine dewatering operations. The research results presented will permit to determine the existing regimes of flow and their extension around pumping wells, trenches and galleries, and the behavoiour of each type of structure by appling the corresponding equation. They provide an important theoretical and practical information that can be utilized for the sizing and construction of water table drawdown systems.

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List of Figures

Figure 1: Trench penetrating the full thickness of a confined aquifer

- h/ Piezometric head at distance x
- h_g/ Piezometric head at the face of the trench
- h_o/ Piezometric head without abstraction
- m/ Aquifer's thickness
- x/ Distance from the face of the trench
- x_o/ Distance from the face of the trench to the place with zero drawdown
- S_g/ Drawdown at the trench

Figure 2: Trench penetrating the full saturated thickness of an unconfined aquifer

- h/ Saturated heigth at distance x
- x/ Distance from the face of the trench
- h_g/ Depth of water inside the trench

Figure 3: Trench penetrating only the upper part of a deep aquifer

- a/ Depth of water inside the trench
- x_1, x_2/ Distances from the center of the trench
- y_1, y_2/ Distances from the free surface of water in the trench to the phreatic level at x_1 and x_2 respectively
- W/ Trench's width

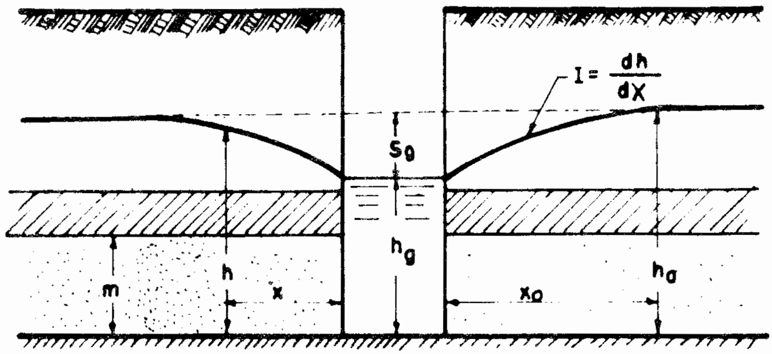

FIG. 1

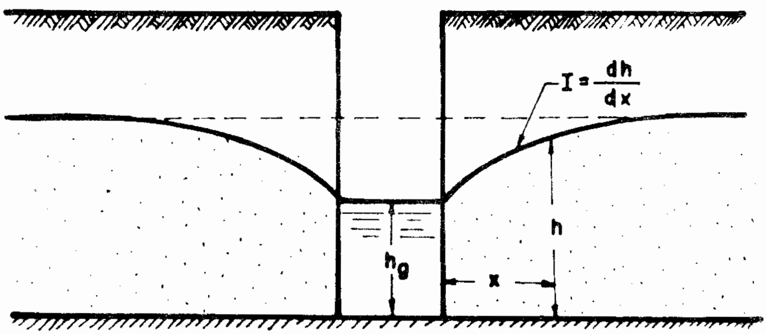

FIG. 2

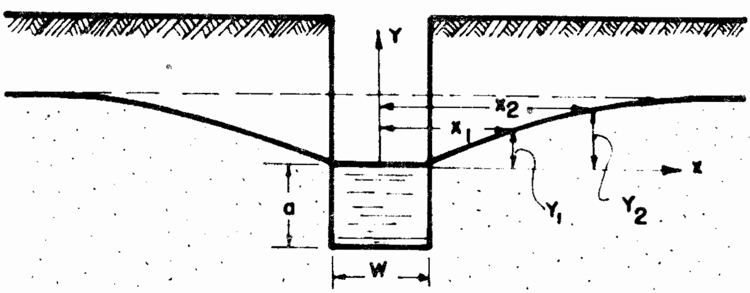

FIG. 3