Designing multi-layer linings of circular openings in watered transversely isotropic rock

N.N.Fotieva, N.S.Bulychev, K.E.Zalessky

Tula State University, Lenin Avenue 92 , Tula, Russia

ABSTRACT

The analytical method of designing multi-layer linings of circular openings in the transversely isotropic medium simulating naturally anisotropic solid rock or the rock weakened by a double-periodic chink set upon the action of the external underground water head is proposed in the paper presented.

MATHEMATICAL MODELLING AND METHOD OF MULTI-LAYER TUNNEL LINING ANALYSIS

The method proposed is based on mathematical modelling the interaction of the underground structure and the surrounding rock mass as elements of a united deformable system undergoing the action of the external ground water pressure.

For the analysis of the lining stress state the corresponding elasticity theory plane contact problem is considered the design scheme of which is given in Figure 1.

The lining is simulated by the multi-layer ring consisting of n layers having the R_i $(i = 1,\dots,n)$ internal radii, isotropic materials of which are characterised by the E_i $(i = 1,\dots,n)$ deformation modules and the ν_i ($i = 1,\dots,n$) Poisson's ratios, and supporting the opening of the R_0 radius in a transversely isotropic S_0 medium characterised by the $E_{0,1}$, $E_{0,2}$ deformation modules correspondingly in the plane of isotropy and in the direction normal to that plane; the $\nu_{0,1}$, $\nu_{0,2}$ similar Poisson's ratios and the $G_{0,2}$ shear modulus in planes normal to the isotropic one. The plane of isotropy may be inclined by an arbitrary β angle with respect to the horizontal one (the Ox axis is located in the plane of isotropy, the Ox^* axis lies in the horizontal plane).

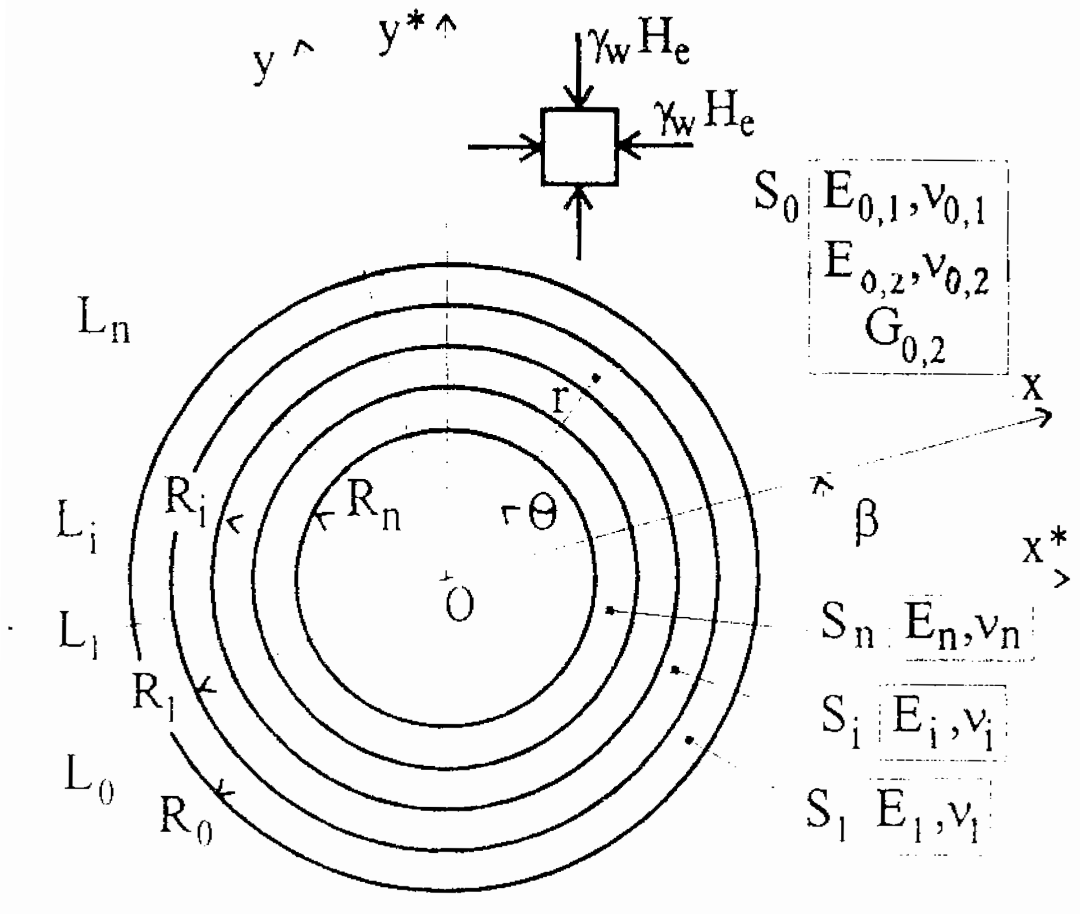

Figure 1: The design scheme.

The transversely isotropic S_0 medium simulates the naturally anisotropic solid rock or the rock weakened by a double-periodic chink set. In the latter case the deformation characteristics of the equivalent transversely isotropic medium may be determined on the testing base (Zelensky, 1969) or by analytic method (Erzanov & Kaydarov, 1970).

The external water pressure is simulated by a presence in the S_0 medium the initial stresses expressed by formula

$$\sigma_x^{(0)(0)} = \sigma_y^{(0)(0)} = -\gamma_w H_e \tag{1}$$

where γ_w is water unit weight, H_e is head of underground water counted off from the tunnel axis.

The S_0 medium and the S_i ($i = 1,\ldots,n$) ring layers undergo deformation together, i.e. conditions of displacements and complete stresses vectors continuity are satisfy on the L_i ($i = 0,\ldots,n-1$) contact lines. The L_n internal outline is free from loads.

Complete stresses in the S_0 medium simulating the rock mass are represented as sums of the initial stresses determined by formula (1) and the additional ones caused by the presence of the opening weakening the massif. The u_x and u_y displacements are considered only as additional ones.

The corresponding contact problem of the elasticity theory is solved with the application of the complex variables analytic functions theory.

Additional stresses and displacements in the S_0 transtropic area are expressed using $\Phi_j(z_j)$ $(j=1,2)$ Lekhnitsky complex potentials by the formulae (Kosmodamiansky, 1976):

$$\sigma_x^{(0)} = 2\,\mathrm{Re}\sum_{j=1}^{2}\mu_1^2\Phi'_j(z_j),\quad \sigma_y^{(0)} = 2\,\mathrm{Re}\sum_{j=1}^{2}\Phi'_j(z_j),\quad \tau_{xy}^{(0)} = -2\,\mathrm{Re}\sum_{j=1}^{2}\mu_j\Phi'_j(z_j)$$

$$u_x^{(0)} = 2\,\mathrm{Re}\sum_{j=1}^{2}p_j\Phi_j(z_j),\quad u_y^{(0)} = 2\,\mathrm{Re}\sum_{j=1}^{2}q_j\Phi_j(z_j), \tag{2}$$

where

$$z_j = x+\mu_j y = x_j + iy_j,\quad \mu_j = i\beta_j,\quad x_j = x,\quad y_j = \beta_j y. \tag{3}$$

Here the μ_j $(j=1,2)$ being parameters of anisotropy are the characteristic equation roots calculated from formula

$$\mu_{1,2}^2 = A(1\mp\sqrt{1-B/A^2}) \tag{4}$$

where

$$A = \frac{\nu_{0,2}}{1-\nu_{0,1}} - \frac{E_{0,1}}{2G_{0,2}(1-\nu_{0,1}^2)},\qquad B = \frac{E_{0,1}-E_{0,2}\nu_{0,2}^2}{E_{0,2}(1-\nu_{0,1}^2)}. \tag{5}$$

For most of the rocks the characteristic equation roots may be adopted as purely imaginary as it has been made in the (3) formulae.

The p_j, q_j $(j=1,2)$ parameters included in the (2) formulae are expressed by the following relationships:

$$p_j = b_{11}\mu_j^2 + b_{12} = -b_{11}\beta_j^2 + b_{12},\quad q_j = b_{12}\mu_j + b_{22}/\mu_j = i(b_{12}\beta_j - b_{22}/\mu_j) \tag{6}$$

where

$$b_{11} = \frac{1-\nu_{01}^2}{E_{0,1}},\quad b_{12} = -\frac{\nu_{0,2}(1+\nu_{0,1})}{E_{0,1}},\quad b_{22} = \frac{1}{E_{0,2}}(1-\frac{E_{0,2}}{E_{0,1}}\nu_{0,2}^2) \tag{7}$$

Additional stresses and displacements in the $S_i(i=1,\ldots,n)$ ring layers (the initial stresses are absent there) are expressed with the $\varphi_i(z)$ and $\psi_i(z)$ $(i=1,\ldots,n)$ Kolosov - Muskhelishvili potentials (Muskhelishvili, 1966). The boundary conditions for determining additional stresses and displacements taking (1) into account are the following:

$$\varphi_1(t) + t\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = (1-\beta_1)\Phi_1(t_1) + (1-\beta_2)\Phi_2(t_2) + (1+\beta_1)\overline{\Phi_1(t_1)} + (1+\beta_2)\overline{\Phi_2(t_2)} - \gamma_w H_e t \quad \text{on } L_0 \tag{8}$$

$$\left[\chi_1\varphi_1(t) - t\overline{\varphi_1'(t)} - \overline{\psi_1(t)}\right]/2G_1 = (b_{12} - b_{11}\beta_1^2)\left[\Phi_1(t_1) + \overline{\Phi_1(t_1)}\right] + (b_{12} - b_{11}\beta_2^2)\left[\Phi_2(t_2) + \overline{\Phi_2(t_2)}\right] + (b_{22}/\beta_1 - b_{12}\beta_1)\left[\Phi_1(t_1) - \overline{\Phi_1(t_1)}\right] + (b_{22}/\beta_2 - b_{12}\beta_2)\left[\Phi_2(t_2) - \overline{\Phi_2(t_2)}\right] \quad \text{on } L_0 \tag{9}$$

$$\varphi_{i+1}(t) + t\overline{\varphi_{i+1}'(t)} + \overline{\psi_{i+1}(t)} = \varphi_i(t) + t\overline{\varphi_i'(t)} + \overline{\psi_i(t)} \quad \text{on } L_i (i=1,\ldots,n) \tag{10}$$

$$\chi_{i+1}\varphi_{i+1}(t) - t\overline{\varphi_{i+1}'(t)} - \overline{\psi_{i+1}(t)} = \frac{G_{i+1}}{G_i}\left[\chi_i\varphi_i(t) - t\overline{\varphi_i'(t)} - \overline{\psi_i(t)}\right] \quad \text{on } L_i (i=1,\ldots,n) \tag{11}$$

$$\varphi_n(t) + t\overline{\varphi_n'(t)} + \overline{\psi_n(t)} = 0 \quad \text{on } L_n \tag{12}$$

where

$$G_i = E_i / 2(1+\nu_i), \quad \chi_i = 3 - 4\nu_i \quad (i=1,\ldots,n) \tag{13}$$

$t = R_0\sigma$ is affix of a point of the L_0 contour in the (8), (9) conditions and $t = R_i\sigma$ $(i=1,\ldots,n)$ on the $L_i (i=1,\ldots,n)$ contours in the (10)-(12) conditions; t_1, t_2 are affixes of points of contours $L_{0,j}(j=1,2)$ restricting infinite $S_{0,j}(j=1,2)$ areas of the $\Phi_j(z_j)$ (j= 1,2) potentials definition obtained from the S_0 area by the (3) affine transformations determined by formulae (Lekhnitsky, 1977):

$$t_j = 0.5R_0\left[(1+\beta_j)\sigma_j + (1-\beta_j)/\sigma_j\right] \quad (j=1,2) \tag{14}$$

It is known that

$$\sigma = \sigma_1 = \sigma_2 = e^{i\theta} \tag{15}$$

The potentials mentioned above are made vanish on infinity and represented on the L_0 outline by the following series:

$$\Phi_j(\sigma) = \sum_{k=1}^{\infty} c_k^{(j)(0)}\sigma^{-k} \quad (j=1,2) \tag{16}$$

The $\varphi_i(z), \psi_i(z)$ ($i = 1,...,n$) complex potentials are regular in the S_i ($i = 1,...,n$) ring layers and are expressed by the Loran series:

$$\varphi_i(z) = \sum_{k=1}^{\infty} c_k^{(1)(i)} \left(\frac{z}{R_{i-1}} \right)^{-k} + \sum_{k=0}^{\infty} c_k^{(3)(i)} \left(\frac{z}{R_{i-1}} \right)^{k},$$

$$\psi_i(z) = \sum_{k=1}^{\infty} c_k^{(2)(i)} \left(\frac{z}{R_{i-1}} \right)^{-k} + \sum_{k=0}^{\infty} c_k^{(4)(i)} \left(\frac{z}{R_{i-1}} \right)^{k} \quad (17)$$

Then expressions (16) and (17) are substituted in the (8) - (11) boundary conditions. The coefficients at the same degrees of the σ variable in the left and right parts of the equations obtained are equated to each other. It allows the recurrent formulae combining coefficients $c_v^{(j)(i+1)} (j = 1,\ldots,4)$ of expansions of $\varphi_{i+1}(z), \psi_{i+1}(z)$ complex potentials into series with the coefficients $c_v^{(j)(i)} (j = 1,\ldots,4)$ of $\varphi_i(z), \psi_i(z)$ potentials to be obtained and then the $c_v^{(j)(n)} (j = 1,\ldots,4)$ coefficients through the $c_v^{(j)(0)} (j = 1.2)$ coefficients of $\Phi_j(\sigma)$ Lekhnitsky potentials to be expressed.

Substituting expressions obtained into the (12) boundary condition we come to an infinite system of linear algebraic equations relative to the unknown $c_v^{(j)(0)} (j = 1,2)$ coefficients. On the above system being solved the stress state of the lining layers is determined.

EXAMPLES OF THE DESIGN

The example of designing tunnel lining upon the external water pressure is given below. The double-layer lining including the external concrete layer with $R_0 = 2.90\text{m}$, $R_1 = 2.42\text{m}$ outer and inner radii correspondingly and the internal steel layer having $R_2 = 2.40\text{m}$ inner radius supports the circular tunnel located in transversely isotropic rock weakened by the double-periodic horizontal chink set characterised by the $\frac{\omega}{a} = 2.5$ relation (here ω is distance between centres of cracks, $2a$ is the length of cracks). The characteristics of the rock and the lining layers materials are the following: $E_{0,1} = 10740\text{MPa}$, $E_{0,2} = 5230\text{MPa}$, $\nu_{0,1} = 0.413$, $\nu_{0,2} = 0.198$, $G_{0,2} = 1200\text{MPa}$; $E_1 = 23000\text{MPa}$, $\nu_1 = 0.2$, $E_2 = 210000\text{MPa}$, $\nu_2 = 0.3$.

The characteristics of the equivalent transversely isotropic solid medium determined according to the work by Erzanov, Aitaliev & Masanov (1980) are: $E_{0,1} = 10740\text{MPa}$, $E_{0,2} = 1480\text{MPa}$, $\nu_{0,1} = 0.413$, $\nu_{0,2} = 0.198$, $G_{0,2} = 450\text{MPa}$.

The distributions of the $\sigma_r^{ex} / \gamma_w H_e$, $\sigma_r^{in} / \gamma_w H_e$ radial contact stresses and the $\sigma_\theta^{ex} / \gamma_w H_e$, $\sigma_\theta^{in} / \gamma_w H_e$ normal tangential stresses appearing on the external and internal outlines of the lining concrete layer are given by solid lines in Figure 2,a,b and in Figure 3,a,b correspondingly.

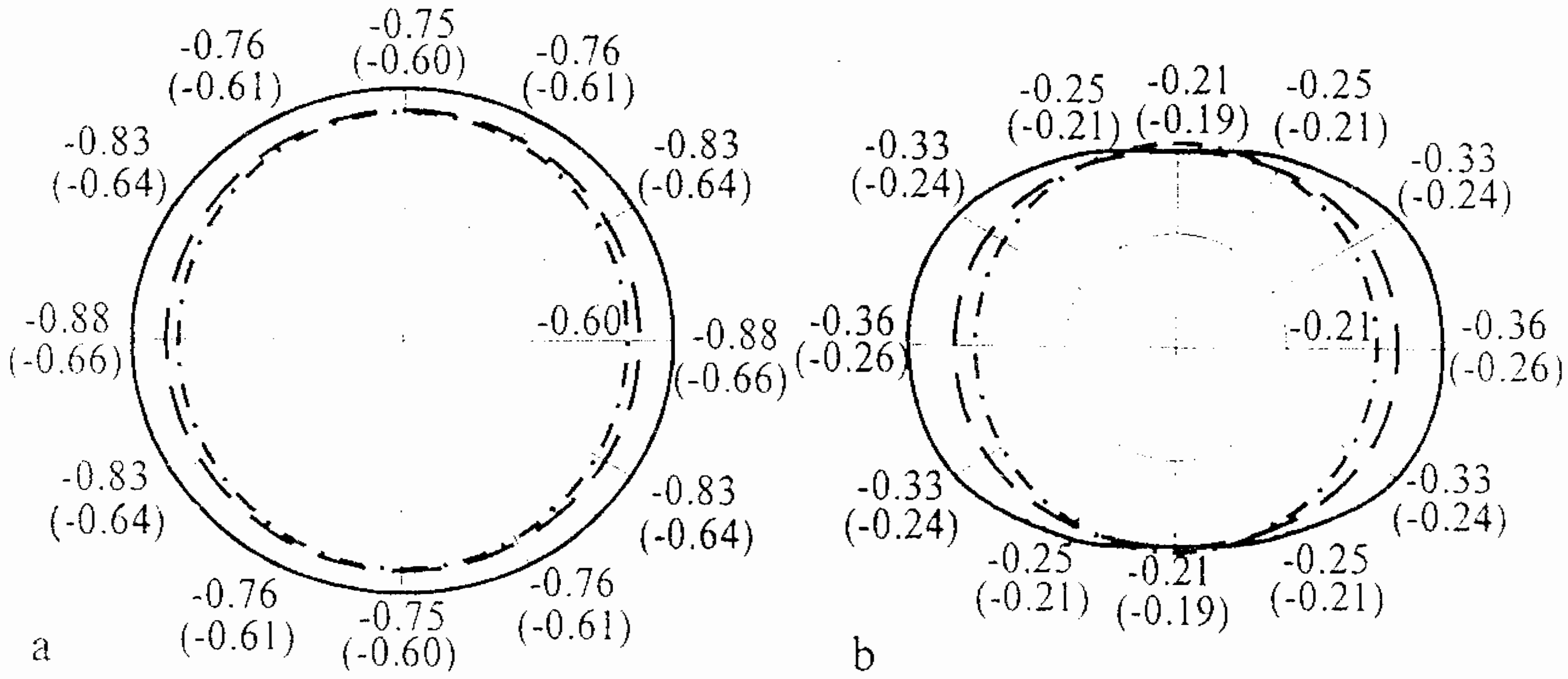

Figure 2: Distributions of radial stresses on the external (a) and internal (b) outlines of the concrete layer.

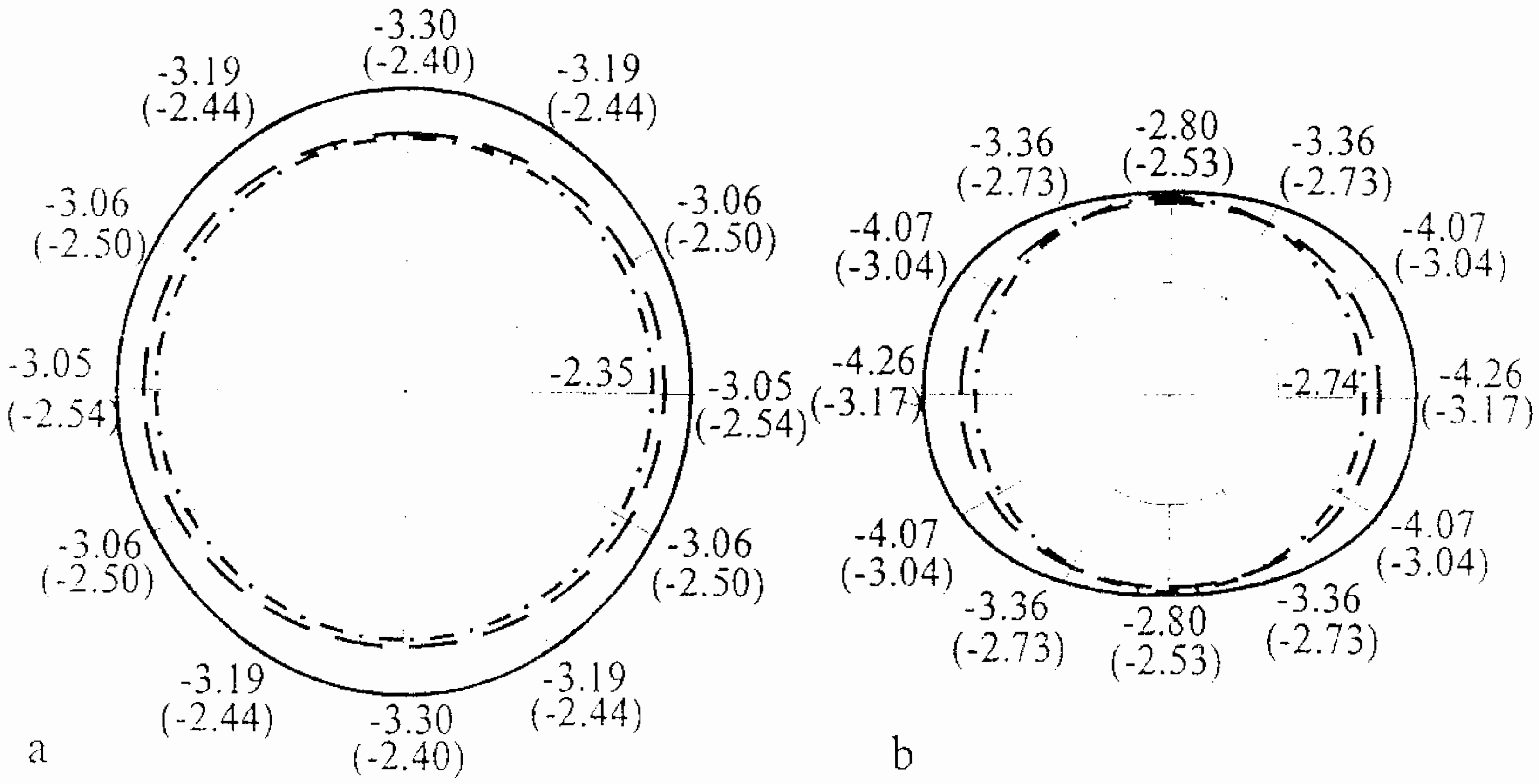

Figure 3: Distributions of normal tangential stresses on the external (a) and internal (b) outlines of the concrete layer.

The distributions of the $\sigma_\theta^{ex} / \gamma_w H_e$, $\sigma_\theta^{in} / \gamma_w H_e$ normal tangential stresses on the external and internal outlines of the lining steel layer are shown by solid lines in Figure 4,a,b.

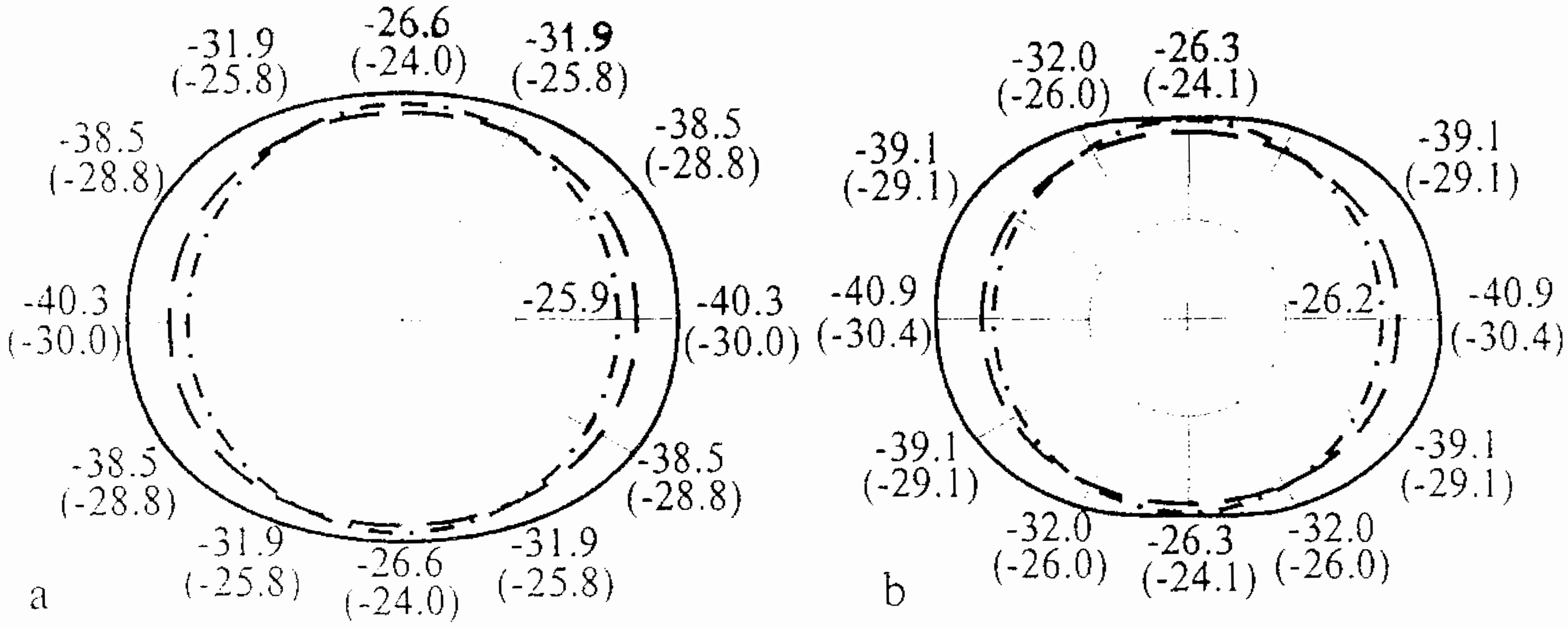

Figure 4: Distributions of normal tangential stresses on the external (a) and internal (b) outlines of the steel layer.

For comparison the corresponding diagrams of the stresses appearing in the lining located in the solid transversely isotropic rock without cracks (the values of stresses are given in brackets) and in the isotropic rock with $E_0 = 5230\text{MPa}$, $\nu_0 = 0.198$ are given in Figures 2-4 by dotted and dot-and-dash lines correspondingly.

REFERENCES

Erzanov, Z.S., Aitaliev, Sh.M., & Masanov, Z.K. 1980. Seismic stress state of underground structures in anisotropic layered massif. Alma-Ata: Nauka (in Russian).

Erzanov, Z.S. & K.K.Kaydarov 1970. On viskoelastic behaviour of massif with a double-periodic chink set. Proc. of the Kazakhstan Academy of Sci. 3:12-18.

Kosmodamiansky, A.S. 1976. Stress state of the anisotropic medium with openings. Kiev: Vysshaja Shkola (in Russian).

Lekhnitsky, S.G. 1977. Elastic theory of anisotropy body. Moskow: Nauka.

Muskhelishwili, N.I. 1966. Certain fundamental problems of mathematic theory of elasticity. Moskow: Nauka (in Russian).

Zelensky, B.D. 1969. Analitic method of evaluation of deformable properties of rock massifs. Hydrotechnic Construction. 11:31-35.