EFFECT OF FRACTURE PERMEABILITY ON CONNECTIVITY OF FRACTURE NETWORKS

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ABSTRACT

Many open pit mines are located in fractured rock systems where water flow paths are complex and difficult to predict. These flow paths are typically controlled by a small subset of fractures that are permeable and interconnected. Most models of flow in fractured rock systems are based on a network of interconnected fractures that are all assumed to be permeable. However this assumption is rarely observed in natural rocks where a significant fraction of the fractures within a connected cluster could be impermeable. Thus in studying fracture flow systems, we need to consider the permeability status (i.e. permeable or impermeable) of individual fractures in addition to the fracture network's connectivity. Primary percolation clusters based on connectivity alone can be generated according to the fracture density, and probability density functions of fracture length and fracture orientation. These primary clusters, potentially including impermeable clusters, may not all conduct water. Hence percolation clusters need to be refined so that they comprise only open fractures. The density of these refined clusters can then be linked to the hydraulic conductivity, providing a more realistic representation of the natural system. Here we use numerical simulations to examine the effect (on connectivity and permeability) of removing a portion of fractures that are assumed to be impermeable. A discrete fracture network model is applied to formulate an analytical relation between two potentially measurable quantities of fractured rock systems, i.e., scan-line density of all fractures within core samples or boreholes and scan-line density of conductive fractures intercepted by boreholes.

1. INTRODUCTION

High pore water pressure and uncontrolled erratic high inflow are two of many challenges that geotechnical engineers are increasingly facing in fractured rocks as excavations, tunnelling and open cast mining deepen, progressing faster and taking place in more demanding rock conditions. Project risks, especially those associated with cost and work safety, are among others strongly dependent on the effectiveness of controlled rock depressurisation and in-pit water management.

The effectiveness of rock depressurisation bores will depend among others: on the likelihood that depressurization bores penetrate highly permeable discontinuities; and on the connectivity of a network of permeable discontinuities. Generally only sparse data of these fundamental properties are available and as (Faybishenko et al., 2000) stated: "*Among the current problems that hydrogeologists face, perhaps there is none as challenging as the characterization of fractured rock".*

Numerical modelling of groundwater flow in fractured rocks is increasingly employed to develop effective depressurisation strategies and to achieve optimal design of depressurisation measures for varying rock characteristics within a project site. Discrete Fracture Network (DFN), Stochastic Continuum (SC) and Channel Network (CN) models have been used with some successes in modelling depressurisation of fractured rock (Dershowitz et al., 1999; Gylling B. et al., 2004; Selroos et al., 2002) However, predicting flow in a fractured rock mass based on structural rock mass parameters is still fraught with a large degree of uncertainty (Neuman, 2005). In particular the fracture aperture distribution cannot be based on direct measurements. Yet the fracture flow is controlled by the size of fracture aperture *(h)* as defined by the cube relation between transmissivity *(T)* and aperture ($T \alpha h^3$) (Kim et al., 2003). The difficulty in measuring fracture aperture seriously constrains the applicability of the DFN concept which (although more appealing in modelling fracture flow) is based on explicit measurements of fracture parameters such as aperture, length and orientation.

Field observations have shown that only a small proportion of fractures contribute to the overall flow, resulting in a complex and heterogeneous flow system. Up to 20% of total number of the fractures may contribute to overall flow (Bear et al., 1993). Although fracture connectivity has been used to explain the heterogeneous phenomenon (de Marsily, 1985), it is likely that additional aspects such as the effect of partial or total closure of individual fractures could further increase flow heterogeneity and tortuosity. Effectively impermeable fractures (although mappable) will not form part of the flow pattern and will thus need to be excluded from the conductive fracture cluster.

The effect of neglecting impermeable fractures on fracture connectivity and subsequent impact on the statistical distribution of rock mass hydraulic conductivity is the focus of this paper and is examined through theoretical Monte Carlo simulations.

2. APPROACH AND METHODOLOGY

The purpose of this study is to examine the effect of excluding impermeable fractures from an interconnected set that spans a model domain. The study is thus based on the percolation theory. This theory examines whether a system has interconnected fractures that span the domain based on the fracture density as well as probability density functions of fracture length and orientation. A key question has been related to the likelihood of a fracture belonging to a percolating cluster. In this study, we aim to demonstrate that an equally important question is: what is the likelihood of tracing a continuous string of permeable fractures that spans the model domain? The percolating cluster would comprise only permeable fractures (with the exclusion of impermeable ones).

To highlight the significance of this question and also to provide an explanation for the heterogeneous nature of fracture flow systems, a series of Monte Carlo simulations were run where in one case all the fractures are assumed to be permeable and in the other varying percentages of impermeable fractures are removed. The distribution and removal of impermeable fractures are assumed to be random and uncorrelated to the fracture length or orientation. The simulation results were then used to examine the relation between the density of a percolating cluster (comprising open and closed fractures) and open percolating cluster (comprising only open fractures). This is similar to examination of field measurements where two sets of data may be available: one comprising of overall fracture density (obtained from borehole core logs or borehole wall imagery) and the other comprising of density of conductive fractures (obtained from flow meters such as heat pulse).

Although we realise that natural fractures would have preferred orientations that are a function of historic stresses, we start our analysis with the simple assumption of fractures being randomly oriented. This allows us to develop the concepts so that in the subsequent papers, we could test these concepts on more realistic scenarios.

Many percolation concepts used in the present study have been well documented in the literature (Sahimi, 1994; Stauffer and Aharony, 1994) and hence are only briefly described here. The percolation theory is perhaps most suited to conceptualise and simulate flow in a fractured rock system since it depicts two fundamental aspects of the fracture flow. These aspects are: (1) fracture connectivity that is essential to ensure the transmission of a fluid across the entire span of the system and (2) heterogeneous and tortuous flow behaviour that is associated with scale dependent hydraulic properties of fractured systems. The essence of the percolation theory is that above a critical fracture density, interconnected fractures form a cluster that spans the model domain (Sahimi, 1994; Stauffer and Aharony, 1994). Near this critical density some macroscopic properties of the interconnected set, such as the hydraulic conductivity, are known to be characterised by a power law distribution. Moreover, at this phase the system's properties are scale-invariant (Sahimi, 1994; Stauffer and Aharony, 1994).

Conductivity and other related properties will be affected by the percolation probability. We define the percolation probability as the chance that a cluster spans from one end of the domain to the other, and it is a function of the fracture density. Since percolation probability is a function of fracture density, for low fracture densities the percolation probability is close to zero and increases up to one as the density increases. There is an 'S' shaped smooth transition between these two states and hence the spanning function can be modelled as a sigmoid function like the modified logistic function (Gershenfeld, 1999):

$$P(d_p) = \frac{1}{1 + a\exp[b(d_c - d_p)]} \quad (1)$$

where $P(d_p)$ is a function that relates the probability of having a percolating cluster with the density; parameters a and b are centre and spread of the cumulative distribution function respectively.

To account for individual fracture permeability, it was assumed that a fracture exists in either permeable (open) or impermeable (closed) state. This assumption, although somewhat simple, is considered to be appropriate for two-dimensional fracture models and necessary because not all fractures are open or permeable. Out of a fracture network some fractures may be rendered impermeable by, for instance, precipitation (Sausse et al., 2001) or due to their orientation in relation to the stress field (Barton et al., 1995). Accounting for the permeability status and ensuring that only open fractures are used to develop a percolating cluster means that: it may be more difficult to obtain a percolating cluster since the fracture density is reduced and also heterogeneity may be more pronounced since the connection is based on reduced fracture density. This reduction of fracture density will have a direct impact on the fracture system's effective hydraulic conductivity. In order to examine the effect of removing some fractures, multiple Monte Carlo simulations were run and percolating clusters of either a mix (closed or open) or only open fractures were generated.

Each simulation scenario is based on 100 Monte Carlo realisations of fracture networks generated on 20×20 m^2 space. The initial simulations are based on varied scan-line densities that may be derived from a typical borehole log fracture frequency data, including both open and closed fractures. For each realisation, a percolating cluster is generated by finding fractures that are interconnected across the length of the domain. The whole algorithm will take a time proportional to N^2 with N being the number of fractures. Subsequently, portions (10-60%) of fractures assumed to be closed are removed. Based on the reduced (open) fracture system, percolating clusters are also generated. For both cases, the scan-line densities of the percolating clusters are determined. Note that the scan-line density is defined as the number of fractures that intercept a unit length of a line drawn across the domain. Vertical scan-lines are drawn across the domain at 5m intervals so that, after each realisation, average scan line fracture densities can be determined.

3. RESULTS

Figure 1 shows an example of a two-dimensional model of randomly distributed fractures where 60% of the fractures are assumed to be open. The orientation of the lines follows a uniform distribution between 0 and 360 degrees and the length l is given by a power law distribution of the form (Walsh and Watterson, 1993):

$$N = cl^{-D} \qquad (2)$$

where N is a cumulative number of fractures with fracture length greater or equal to l; c is a constant and D is a power exponent.

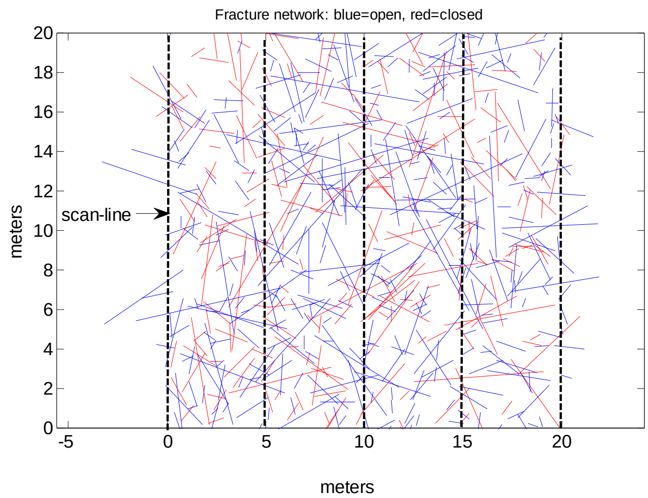

Figure 1. A fractured rock network generated with the simulation code. The line segments represent fractures with the blue ones being those that are permeable (open) and red being those that are impermeable (closed). In this realisation 60% of the simulated fractures are open. The average scan line density is 1.45 fractures/meter (fr/m). The fracture length follows a power law distribution with a minimum length of 0.5 m and a maximum of 8 m, and the power exponent of 1.6. The orientation distribution is uniform between 0 and 360 degree. Vertical dashed lines show the scan-lines.

Once a portion of open fractures is determined the effect of density of either total fractures or only those that are open on percolating cluster is examined, as shown in **Figure 2**. Insert figure (a) shows the relation between scan-line density of a percolating cluster and total scan-line density (i.e all fractures some closed and others open) and insert figure (b) shows the scan-line density of open percolating cluster and that of open fractures. In both plots it is evident that a non-linear relation exists at lower densities and becomes linear as density increases. A visual inspection of **Figure 2** shows that the two plots are similar. The reduction in fracture density is characterised by a longer tail (for open fractures in plot (b)) where the probability of percolation is close to zero. This highlights the effect of accounting for permeability status of each and every fracture and basing the percolating cluster only on open or permeable fractures. The effect is that for some clusters, even though an interconnection of fractures across the domain may exist effective permeability could be zero. This is because the connectivity would, in part, be directly controlled by closed fractures.

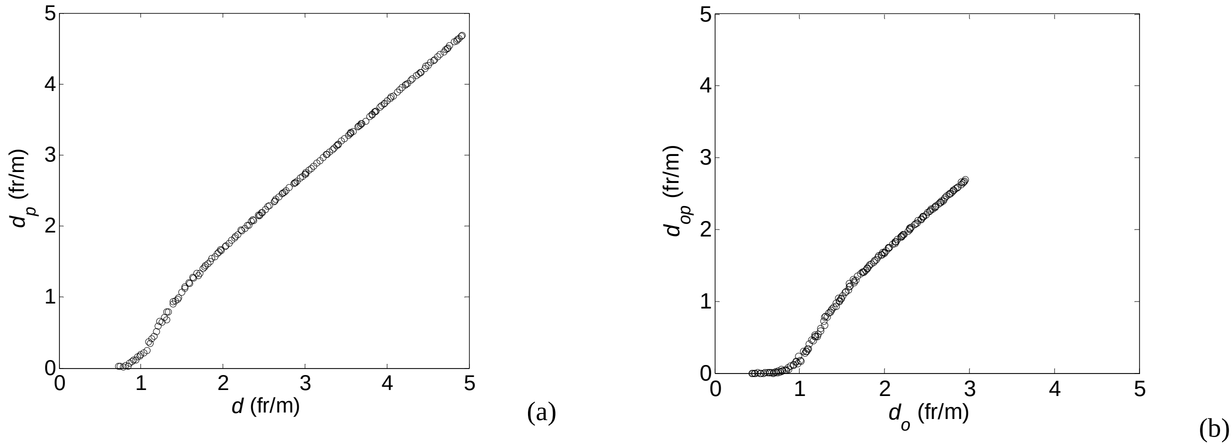

Figure 2. Relations between the scan-line densities. (a) shows relation between density of percolating cluster and total fracture density and (b) shows resultant density of open percolating cluster and density of open fractures.

The effect of removing a portion of the fractures from the total density can be conceptualised in **Figure 3.** The realisation starts with a total fracture scan-line density *(d)* (that comprises a mixture of open and closed fractures) that can produce a percolating cluster *(d_p)*. A fraction (εd) of the total fracture density represents a set of open or permeable *(d_o)* fractures only. Out of this fraction of fractures it is possible that a percolating cluster *(d_{op})* may be generated. As already mentioned **Figure 2** has shown similarity between plots 2(a) and 2(b). It can thus be inferred that functions that relate d and d_p and also d_o and d_{op} are similar. It is easy to test this by superimposing the two plots **Figure 4**

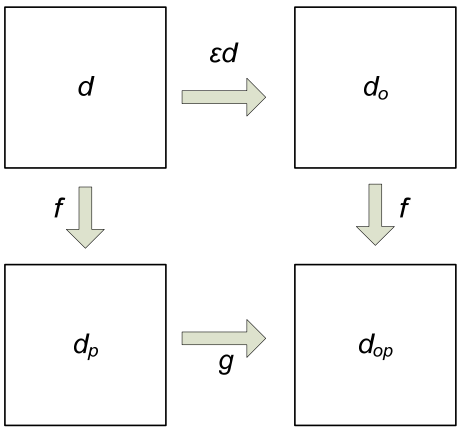

Figure 3. Relation between the different scan-line densities.

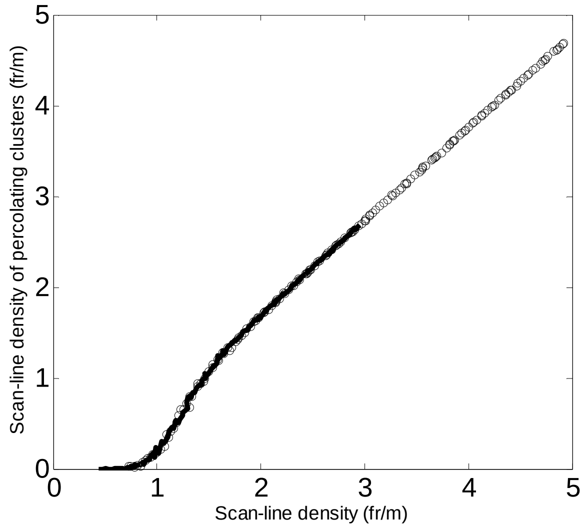

Figure 4. Plots of d_p vs d (circle) and d_o vs d_{op} (line) exactly superimposed and showing that they are defined by the same function.

We then examined the relationship between d_p and d_{op}. As shown in **Figure 5**, we can see that when 10% of the fractures were assumed to be open, the simulated fracture density range was not high enough to ensure a percolating cluster that comprised solely open fractures. For a 20% likelihood of fractures being open, a relatively high fracture density (>2.5 fractures/m) was required before percolating open clusters could develop. The 30% proportion of open fractures seems to be a critical condition for the formation of open percolating clusters.

In practice a proportion of fractures that are conductive may be used to determine the scan-line density of open percolating cluster. In particular, for a saturated fractured rock mass it would be reasonable to assume that the average scan line density of conductive fractures can be used as average density for open percolating cluster. A conductive fracture is expected to be: (i) open and (ii) belong to a percolating cluster that is connected to a fluid source.

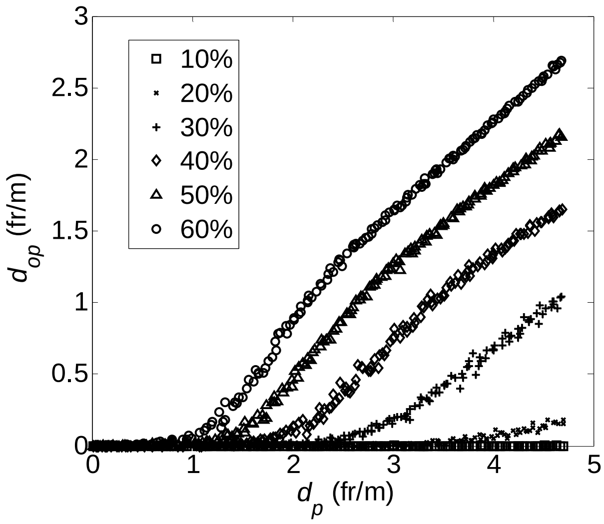

Figure 5. Relation between scan-line densities of open percolating cluster d_{op} (i.e. comprising only open fractures) and percolating cluster d_p (i.e. comprising open and closed fractures). Different symbols represent different percentages of open fractures.

Figure 6 provides a closer look into simulations with 60% of the fractures assumed to be open and shows the probability distribution of open percolating clusters when we already have a percolating cluster that comprises open and closed fractures (dp). The probability of percolation affects the average density of open percolating clusters. A non-linear relation below 100% percent probability exists and becomes linear at 100% probability. The spanning probability function P(dp) defined in the percolation theory literature (Stauffer and Aharony, 1994) (explained by equation (1)) was used by (Skvor and Nezbeda, 2009) to justify a logistic function to fit the percolation probability in a similar problem. This function (P(dp)) fitted with equation (1) is shown in Figure 6 and shows a good fit with the obtained data.

The combined effects of the linear relation at 100% probability and the nonlinear behaviour below this point can be represented by the product of a linear function and the spanning probability function as shown below:

$$d_{op} = g(d_p) = \varepsilon(d_p - d_c)P(d_p) \qquad (3)$$

Where d_c is critical scan-line fracture density at which open percolating cluster forms; $P(d_p)$ is the probability of generating an open percolating cluster from an existing percolating cluster of open and closed fractures. The validation of this model is shown in **Figure 7**. As expected the model indicates that a non-linear relation exists at low densities and changes to a linear relation as the density increases. In fact the slope of the linear section is given by the ratio of density of open fractures to that of total fractures and, for example, is 0.6 when 60% of the fractures are assumed to be open. Additionally, the fraction (ε) of open fractures controls the parameters *a* and *b*.

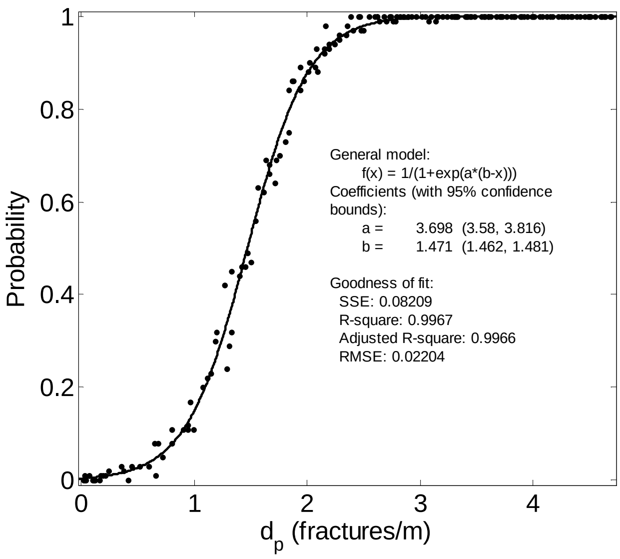

Figure 6. Percolating probability $P(d_p)$ versus percolating cluster density d_p. Dots are from the simulations and the solid curve results from the fitting of the spanning function to the numerical results.

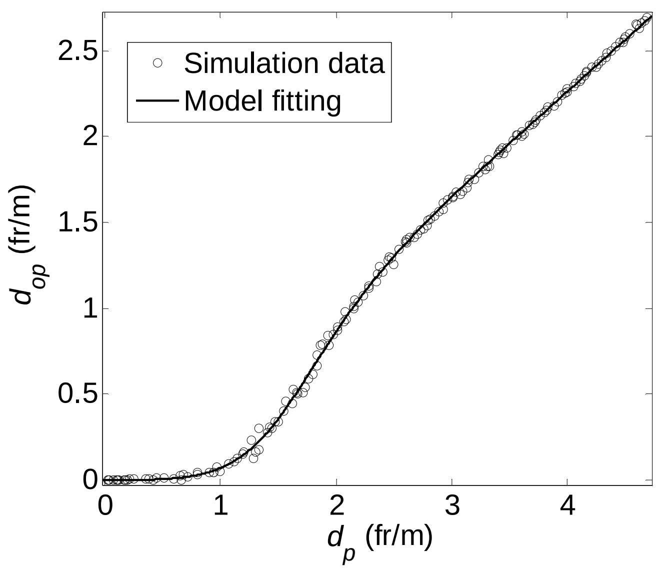

Figure 7. Relation between simulated densities (dots) of percolating cluster d_p and open percolating cluster d_{op} fitted with the model of equation (3) (solid curve).

The analytical model (i.e. equation 3) was found to match well the simulation results for the whole density range. We can use this model to further explore the relation between two scan-line densities that can be possibly measured *viz* density of all the fractures (i.e. via borehole wall imagery, core logs, acoustic or optical televiewer logs) and density of conductive fractures (i.e. from high precision flow logging, e.g. heat pulse flow meter). We recall that the same function *(f)* can be used to describe the relation between d and d_p, and that between d_o and d_{op} (Figure 4). In other words the function $f(d)=d_p$ describing percolating cluster of open and closed fractures is the same as the function $f(d_o)=d_{op}$ that describes the open percolating cluster.

From the relations shown in **Figure 3.** it may also be deduced that the function f is similar to function g. Function f describes the percolation of a mix of fractures (d to d_p) and function g describes the connectivity of a percolating cluster (d_p to d_{op}) when some of its fractures are neglected because they are closed. Thus the difference is that from d to d_p we generate a new percolating cluster whilst from d_p to d_{op} we remove closed fractures and may recover an open percolating cluster. Since we are dealing with different types of percolating processes the parameters of equation (1) (i.e. a and b) would be different. We develop a functional equation for these relations, obtained by describing the density of open percolating fractures as a function of total density:

$$f(\varepsilon d) = g(f(d)) \quad (4)$$

To examine this functional relationship, we compared both composite functions for the case with ε equal to 0.6 as shown in Figure 8.

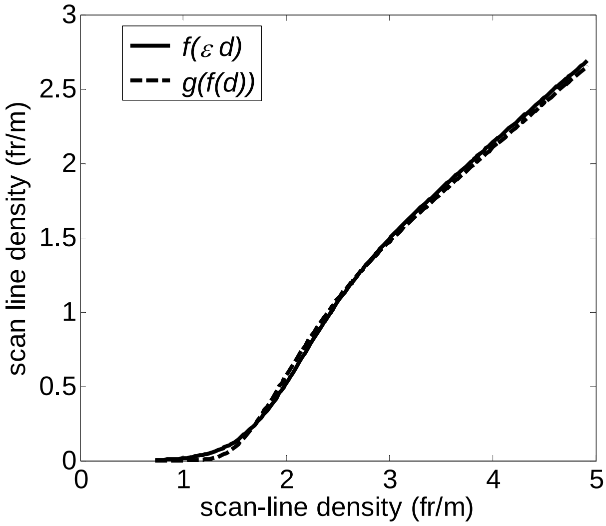

Figure 8. Verification of the functional relation (equation (4).

The superimposed results indicate that *f(εd)* is similar to *g(f(d))*

The two relations are almost identical and hence for analysis we can use the simpler one *f(εd)*, which is almost of the same form as equation (1).

Only once a percolating cluster that comprises solely open fractures has been defined can hydraulic conductivity of the overall fracture system be estimated. The dependence of effective hydraulic conductivity on density of open and percolating clusters is shown in Figure 9. The significance of this is that we are able to express effective hydraulic conductivity as a function of scan-line density.

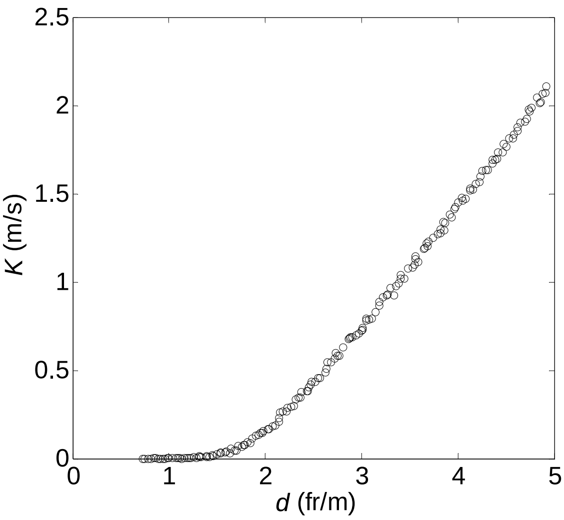

Figure 9. Hydraulic conductivity as a function of the density of the percolating cluster for $\varepsilon = 0.6$.

4. CONCLUDING REMARKS

We have presented a method that is easy to code and yet handles complex fracture networks as well as their properties such as connectivity.

We have used the discrete fracture network model to formulate an analytical relation between two measurable quantities of fractured rock systems, i.e., (i) scan-line density of a fractured rock mass measurable from core samples or with acoustic or optical televiewer and (ii) scan-line density of conductive fractures that can be measured for example with highly sensitive heat pulse flow meters .

Most previous research on the behaviour of connectivity and hydraulic conductivity of fracture systems has focused on the power law scaling behaviour around the percolation threshold. However our model based on equation (1) and scan-line density of percolating clusters is more general and seems to be applicable for all ranges of considered densities.

Further studies are suggested to derive functional relationships between scan-line densities of open and closed fracture cluster for a wider range of fracture networks including fracture networks with bimodal probability function of fracture orientation and for fracture networks showing fracture termination. We further suggest to investigate the use of scan-line fracture densities and the functional relationships proposed in this paper for optimising pit wall depressurisation strategies by means of horizontal depressurisation bores or vertical dewatering bores.

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