**Water abundance prediction method of weathered bedrock based on improved AHP and the entropy weight method**

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**Abstract**

The Jurassic weathered bedrock aquifer in Northwest China is the main aquifer for filling water in coal mines. Many water inrush disasters in the coal mine are related to the aquifer. The south wing of Ningtiaota coalfield was taken as an example to analyze the water abundance of weathered bedrock and its controlling factors. The water-rich zoning of weathered bedrock aquifer is carried out by using AHP and entropy weight method to calculate the weights separately and coupled with each other. The results indicated that coupled of improved AHP and the entropy weight method can be used to predict the water-richness of weathered bedrock accurately.

**Keywords:** AHP, the entropy weight method, weathered bedrock, water abundance, Ningtiaota coalfield

**Introduction**

Northwest China is rich in coal resources and is the main coal production area in China at present and in the future (Chen and Jiang 2007). In recent years, many water inrush disasters of Jurassic weathered bedrock occurred in the mining of Jurassic coal resources in Northwest China, which made people gradually realize the importance of mastering the water richness of Jurassic weathered bedrock in mine water prevention. However, the water abundance of Jurassic weathered bedrock is very uneven in space. How to evaluate and predict the water-rich of it has become an urgent problem to be solved. The evaluation and prediction of aquifer's water-enrichment is an important basic work in mine water disaster prevention and control (Xi et al. 2015). Many scholars have studied it and put forward a multi-factor comprehensive analysis method (Dai et al. 2016; Wu et al. 2011; Hou et al. 2016; Wang et al. 2014), which is on the basis of analyzing the influencing factors of aquifer’s water-enrichment, the quantitative evaluation and prediction model of aquifer’s water-enrichment is established.

It is very important to determine the weight of each factor when a model of aquifer water-rich evaluation and prediction is established. Because different weights have different prediction results for water abundance, and it is often determined by Analytic Hierarchy Process (AHP). However, most of the weights of water-rich influencing factors in AHP are determined by experts according to the 1-9 scale method created by T.L.SAATY, and the weights of each factor are not make full use of the measured data and the calculated weights are subjective (Guo et al. 2008). Zhang et al. (2006) proposed an improved method for determining the weight of AHP indicators. In addition to considering the scoring of experts, the method also considers the standard deviation of the measured data of each index, which improves the application effect to a certain extent. In order to overcome the subjectivity of AHP computing weights and predicted the heterogeneity of the aquifer, the entropy weight method was adopted by Ma et al. (2011). The advantage of this method is an objective evaluation method for determining the weight based on the measured data of...
each index, but the disadvantage is that the expert's experience is not considered when determining the weight of each index. Therefore, there are certain limitations to using either method alone. The south wing of Ningtiaota coalfield in North Shaanxi is used as example, discussing the prediction method of weathered bedrock water abundance by coupling the improved AHP and entropy weight method to determine the index weight. The coupling is through reasonable mathematical methods, which considering both expert experience and measured data characteristics, so that the prediction results of water richness evaluation are more accurately.

**General Situation of the Study Area**

The Jurassic coalfield is one of the main coal producing areas between Shanxi, Shannxi and Inner Mongolia of north China, which coal resources account for about 14% of the national total. The study area is located in south wing of Ningtiaota coalfield in Shenmu City, Shannxi Province. The overlying aquifer is porous phreatic water of Quaternary, fissure confined water in weathered bedrock of Zhiluo Formation and fissure confined water in Yan'an Formation. The fissure confined water in weathered bedrock of Zhiluo Formation is widely distributed in the study area, with an average thickness of 26.62 m. The unit inflow of borehole is 0.1183 L/s.m, which is the main water-filling aquifer and poses more threat to mine safety production.

**Controlling factors of weathered bedrock water abundance**

In order to evaluate water enrichment more effectively, author analyzed 41 pumping tests data of weathered bedrock aquifer in the study area. It was found that the water enrichment of weathered bedrock was mainly controlled by 4 factors:

1. **Top-level index**

   According to the analysis of pumping tests data, there is a certain correlation between the water abundance of weathered bedrock and its top-level. For example, the unit inflow of boreholes BK42 and K3-1 are 0.1051 L/s.m and 0.1220 L/s.m. The corresponding top-level are 1180.40m and 1218.32. In order to clearly express this correlation, the elevation of weathered bedrock is graded according to the height of 15 m. The larger the value, the stronger the water richness.

2. **Lithologic association index**

   According to the data of pumping boreholes, use the contribution of the lithology combination to the water-richness, it is quantified (Table 2). The larger the value, the stronger the water-richness.

3. **Weathered index**

   The stronger the weathering degree is, the better the water-rich is. Quantify rock weathering by category (Table 3). The weathering influence index $w$ is constructed to describe the influence of weathering degree

<table>
<thead>
<tr>
<th>Elevation m</th>
<th>&gt;1240</th>
<th>1225–1240</th>
<th>1210–1225</th>
<th>1195–1210</th>
<th>&lt;1195</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantized value</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lithology category</th>
<th>Mudstone</th>
<th>Sandy mudstone</th>
<th>Siltstone</th>
<th>Fine sandstone</th>
<th>Medium sand</th>
<th>Coarse sandstone</th>
<th>Conglomerate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantized value</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weathering degree</th>
<th>Weak</th>
<th>Medium</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantized value</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
and thickness on water abundance. The larger the value, the stronger the water-rich.

\[ w = \frac{\sum w_i h_i}{\sum h_i} \]  

(2)

In this formula, \( w_i \) is weathered degree of different lithologic rocks, \( h_i \) is thickness of weathered bedrock with different lithology.

4 Core rate index

The lower the value and the better water richness. The core rate of weathered bedrock in each borehole is calculated by constructing core recovery index \( c \), which reflects the water-rich of weathered bedrock from the side. The smaller the value, the stronger the water richness.

\[ c = \frac{\sum c_i h_i}{\sum h_i} \]  

(3)

Where \( c_i \) is rock formation rate; \( h_i \) is strata thickness.

In conclusion, water abundance of weathered bedrock can be predicted by four indexes: top-level index, lithologic association index, weathered index and core rate index. The hierarchical structure model is shown in Figure 1.

Method

Improved AHP indicator weight determination method

The specific content of the improved analytic hierarchy process is as follows: for each of \( n \) index, the standard deviation of samples is calculated separately, then the standard deviation of samples is compared with each other, and the values of other elements of the judgment matrix can be obtained in turn (Huang and Zheng 2003).

The sample standard deviation \( S(i) \) of each evaluation index can be used to reflect the influence degree of each evaluation index on aquifer water-rich, and to construct a judgment matrix \( B_{max} \). The calculation formula of its internal value \( b_{ij} \) is as follows:

\[
\begin{align*}
   b_{ij} &= \begin{cases}
   \frac{S(i) - S(j)}{S_{max} - S_{min}}(b_{m}-1) + 1, & S(i) \geq S(j) \\
   \frac{1}{S_{max} - S_{min}}(b_{m}-1) + 1, & S(i) < S(j)
   \end{cases} \\
   b_{m} &= \min \left\{ 9, \inf \left[ \frac{S_{max}}{S_{min}} + 0.5 \right] \right\}
\end{align*}
\]

In this formula, \( S_{max} \) and \( S_{min} \) are the maximum and minimum values of \( S(i) \) respectively; Relative importance parameter \( b_{m} \), \( \min \) and \( \inf \) are the smallest and integral functions respectively. Finding the eigenvector \( u_j \) corresponding to the maximum eigenvalue \( \lambda_{max} \) according to the matrix.

\[ M_i = \prod_{j=1}^{n} b_{ij} \quad (i = 1, 2, 3, \ldots, n) \]  

(5)

\[ u_j = \frac{u_j}{\sum_{i=1}^{n} u_i} \]  

(6)

Because of the complexity of multi-order judgment matrix and some values in the judgment matrix may be inconsistent. It is necessary to check the consistency of

Fig.1 Hierarchy model for predicting water-richness of weathered bedrock aquifer
2. Entropy of Evaluation Index

In the evaluation questions of n evaluation targets and m evaluation indicators, the entropy of the j index is defined as:

\[ H_j = -\sum_{i=1}^{n} p_i \ln p_i, \quad (j=1,2,3,\ldots,m) \]

In this formula, \( p_i \) is the weight calculated by the entropy weight method.

3. Evaluation index entropy weight

In this paper, the least square method of optimization method is used to couple improved AHP and the entropy weight to form a subjective and objective weighting method.

\[ \text{Method for determining index weights for coupling method} \]

In this paper, the least square method of optimization method is used to couple improved AHP and the entropy weight to form a subjective and objective weighting method.
In this formula, $A$ is a diagonal array. $e$, $W$ and $B$ are $m$ rows and one column vector.

$$A = \text{diag} \left[ \sum_{i=1}^{n} z_{1i}^2, \sum_{i=1}^{n} z_{2i}^2, \ldots, \sum_{i=1}^{n} z_{mi}^2 \right]$$  \hspace{1cm} (17)

$$e = [1, 1, L, 1]^T \quad w_j = [w_1, w_2, L, w_m]^T$$

$$B = \begin{bmatrix} \sum_{i=1}^{n} \frac{1}{2} (u_1 + v_j) z_{1i}^2, \sum_{i=1}^{n} \frac{1}{2} (u_2 + v_j) z_{2i}^2, \ldots, \sum_{i=1}^{n} \frac{1}{2} (u_m + v_j) z_{mi}^2 \end{bmatrix}$$  \hspace{1cm} (18)

Solve the matrix equation above and get $w_j$:

$$w_j = A^{-1} \times \left[ B + \frac{1 - e^T A^{-1} B}{e^T A^{-1} e} \times e \right]$$  \hspace{1cm} (19)

Where $w$ is the weight of the improved AHP and entropy weight method coupling calculation, $Z_{ij}$ is a standardized data matrix with $m$ evaluation indicators and $n$ evaluation objects.

**Prediction of weathered bedrock water abundance in study area**

**Water richness index weight calculation**

According to the data of 172 boreholes in the study area, the weight of water-rich prediction indicators are calculated by improved AHP, entropy weight method and the coupling method.

1. **Index Weight Calculation Based on Improved AHP**

The judgment matrix $B_{4 \times 4}$ calculated according to the formula 4 is as follows:

$$B_{4 \times 4} = \begin{bmatrix} 1 & 4.41 & 5.23 & 9 \\ 0.23 & 1 & 1.81 & 5.58 \\ 0.19 & 0.55 & 1 & 4.77 \\ 0.11 & 0.18 & 0.21 & 1 \end{bmatrix}$$

The judgment matrix is a $4 \times 4$ matrix, which is calculated by formulas (7) and (8). The maximum eigenvalue of the judgment matrix is $\lambda_{\max} = 4.1513$, $CR=0.0348<0.1$. Therefore, the judgment matrix satisfies the consistency test.

The weight of the index layer $u_i$ is obtained by calculation: The weight $u_1$ of top-level index is 0.6197, the weight of lithologic association index $u_2$ is 0.2016, the weight of weathered index $u_3$ is 0.1372, and core rate index $u_4$ is 0.0415(Table 4).

2. **Index Weight Calculation Based on Entropy Weight**

The matrix normalized by Equation 10 as follows:

$$Y = \begin{bmatrix} 0.2500 & 0.2950 & 0.4850 & 0.8939 \\ 0.0000 & 0.5000 & 0.6787 & 0.2641 \\ 0.2500 & 0.5000 & 0.5826 & 0.2643 \end{bmatrix}$$

The normalized matrix is calculated by using formulas (13) and (14) in Matlab, and the weight $v$ of each index is obtained: The weight $v_1$ of top-level index is 0.4136, the weight of lithologic association index $v_2$ is 0.1779, the weight of weathered index $v_3$ is 0.2614, and core rate index $v_4$ is 0.1471(Table 4).

3. **Index Weight Calculation Based on the Coupling Method**

The improved AHP and the entropy weight method are coupled by the least squares method. From equation (17), $A$ is a $4 \times 4$ diagonal matrix, as follows:

$$A = \begin{bmatrix} 51.6875 & 0 & 0 & 0 \\ 0 & 74.5079 & 0 & 0 \\ 0 & 0 & 49.5930 & 0 \\ 0 & 0 & 0 & 51.0274 \end{bmatrix}$$

In matlab, formulas (18) and (19) are used to calculate the optimal weight $w_j$ of the index layer: The weight $w_1$ of top-level index is 0.5166, the weight of lithologic association index $w_2$ is 0.1898, the weight of weathered index $w_3$ is 0.1993, and core rate index $w_4$ is 0.0943(Table 4).
Evaluation and Verification of Water Abundance

According to the standardized data rasterized in Surfer, the grid maps of each index are imported into ArcGIS, then the raster calculator is used. Combined with the improved AHP, the entropy weight method and the weight calculated by coupling, the overlay analysis is carried out respectively. Finally, the zonal prediction maps of weathered bedrock water abundance in the southern wing of Ningtiaota coalfield under different weighting conditions are obtained (Fig 3, Fig 4 and Fig 5). According to the natural discontinuity method, the water abundance is divided into four grades: extremely weak (I), weak (II), medium (III) and strong (IV).

According to the characteristics of unit inflow of 41 pumping holes in this area, the unit inflow of borehole is divided into four categories, namely less than 0.01L/s·m, 0.01–0.1L/s·m, 0.1–1L/s·m and 1–5L/s·m corresponds to the extremely weak (I), weak (II), medium (III) and strong (IV) in Figures 3, 4 and 5 respectively (Table 5).

In order to compare the advantages and disadvantages of three water abundance partition prediction methods based on improved AHP, entropy weight method and the coupling method, the water abundance level of the weathered bedrock pumping

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### Table 4 Weighted prediction index of weathered bedrock calculated by different methods

<table>
<thead>
<tr>
<th>indicators</th>
<th>top-level index of the weathered bedrock</th>
<th>lithologic association index</th>
<th>weathered index</th>
<th>core rate index</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHP weight ($u_j$)</td>
<td>0.6197</td>
<td>0.2016</td>
<td>0.1372</td>
<td>0.0415</td>
</tr>
<tr>
<td>The entropy weight ($v_j$)</td>
<td>0.4136</td>
<td>0.1779</td>
<td>0.2614</td>
<td>0.1471</td>
</tr>
<tr>
<td>Coupling weight ($w_j$)</td>
<td>0.5166</td>
<td>0.1898</td>
<td>0.1993</td>
<td>0.0943</td>
</tr>
</tbody>
</table>

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**Fig.3 Water abundance prediction zoning map based on AHP**

**Fig.4 Water abundance partition prediction map based on the entropy weight method**
borehole in the area is compared with forecast categories in Figures 3, 4 and 5. If they are consistent, they are considered to be the same, indicating that the prediction results are correct. By comparison, it is found that the water abundance classification predicted by the improved AHP method, the entropy method and the coupling method are 68.29%, 70.73% and 92.68% respectively. So the least squares method is the best method for water abundance zoning prediction.

**Conclusion**

The fissure aquifer of weathered bedrock in Jurassic coalfield in northern Shaanxi is the main water-filling aquifer in the mine. Its water abundance varies greatly in space. Four indexes can be used to predict the water richness of weathered bedrock aquifer, top-level index, lithologic association index, weathered index and core rate index.

The water abundance prediction method based on coupling of improved AHP and entropy weight method is a accurate method. It is more effective than the improved AHP and entropy weight method when used alone.

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