SIMPLIFIED MATHEMATICAL MODELS FOR THE CALCULATION OF DEWATERING

by Dr. Sándor Léczfalvy

VIZITERV Budapest 1397 Pf.527.

SUMMARY

The study consists of two parts. One of them discusses the dewatering of the artesian aquifer operated by stratum pressure. The dewatering of the free water surface model is discussed in the other parts.

At the artesian aquifer first of all, the formulas for the calculation formulas of an isolated artesian well in the case of nonsteady flow are derived, dealing first with the case, where the radius of the circle of influence /R/ does not extend to the border of the permeable layer, i.e. to the supply area /R_o/. After this, when the $R=R_o$, other formulas are derived.

At the above cases, when the constant drawdown condition is existing /$s=s_o$/ the $Q=f/t/$ formula /Q=discharge, t time of pumping/; and when $Q=Q_o$=constans, the drawdown function /$s=f/t/$/ are given.

Also for the moment of attaining the border of the supply area /t_v/ a new formula is presented.

The above formulas are justification in the nature.

At the stratum pressure aquifer formulas have been established for the calculation of the time - dependent variation of the depression taking place in each of these wells, in the case of simultaneous operation of a number of artesian wells in a permeable stratum of infinite extension, and of a constant exploitation, finally the discharge series are calculated for a constant drawdown.

The second part of the study presents the formula $s=f/t/$ when $Q=Q_o$=const, at the free water surface.

A./ DEWATERING OF ARTESIAN AQUIFER OPERATED BY STRATUM PRESSURE

I. ANALYSIS OF SINGLE ARTESIAN WELL.

1./ The Discharge of Well Placed in the Infinite Aquifer with Constant-drawdown Condition.

There is an artesian aquifer with the notation of figure 1, in which aquifer of thickness, m, and k coefficient of permeability. There will be constant drawdown condition $/s=s_0=H_0-y_0/$ at the well of r_0 radius. In this condition expressions will be developed for discharge /Q/, and drowdown radius /R/ depending time after pumping started /t/.

At time t the influence range of drawdown /drawdown radius/ is R, and after time increment dt this becomes R+dR. If the discharge during unit time is Q, then the discharge during the time increment dt is Q.dt. This water is in proportion to the change of the piezometric volume /dV/, i.e

$$Qdt = \beta\, dV \qquad /1/$$

According to Fig 1,

$$dV = \beta\, 2R\, \pi\, dR\ /H_0-y_0/ \qquad /2/$$

is a proportionality factor, whose value can be found by pumping /see later/.

And thus

$$Qdt = \beta\, 2R\pi\, dR\ /H_0-y_0/ \qquad /2a/$$

If the formula of Dupuit-Thiem is considered valid at the moment t, that is

$$\frac{2\pi mk(H_0-y_0)}{\ln\frac{R}{r}}\,dt = \beta(H_0-y_0)\,2\pi R\,dR \qquad /3/$$

by rearranging equation /3/

$$\frac{mk}{\beta}\,dt = R\ln R\,dR - R\ln r\,dR \qquad /4/$$

Remembering that for t=0, also R=r and $C=\frac{r^2}{4}$, the differential equation is solved R in the form

$$R=\sqrt{\frac{\frac{2mk}{\beta}t-\frac{r^2}{2}}{\ln\frac{R}{r}-\frac{1}{2}}} \qquad /5/$$

Above can be seen, that R does not depend on the drawdown $/s=H-y_0/$ only the time after discharge started /t/. Equation /5/ involves a process of successive approximation because R is on the right side too.

Discharge has been computed from the approximately formula:

$$Q=\frac{2\pi mk(H_0-y_0)}{\ln\frac{R}{r}} \qquad /6/$$

With different t R can be calculated according to equation /5/, and put into /6/, the Q=f/t/ function can be determined.

2./ Calculation of Drawdown of Artesian Well Situated in the Infinite Aquifer with Constant Discharge Condition /$Q=Q_1$/.

In Eq /6/ $H_0-y_0=s$, If $Q=Q_1$ is constant H_0-y_0 has to be variable. Thus

$$s = H_o - y_o = \frac{Q_1 \ln R\, r}{2\pi mk} \qquad /7/$$

as R depends on t only, its value can be calculated from Eq /5/, and can be put into Eq /7/.

3./ Discharge of Well Situated on Finite Artesian Aquifer With Constant - drawdown Condition.

As the drawdown radius /R/ attained the boundary of the aquifer, it will be constant, i. e $R=R_0$.

In this case with reference to Fig. 2. we study the yield of well with constant-drawdown.

Set out from

$$Q\,dt = -\beta R_o^2 \pi\, dH \qquad /8/$$

that is

$$\frac{2\pi mk(H-y_o)}{\ln R/r}dt = -\beta R_o^2 \pi\, dH \qquad /9/$$

Rearranging equation /9/ and introducing the notation

$$A = \frac{2mk}{R_o^2 \ln R_o/r} \qquad /10/$$

We get

$$\frac{A}{\beta}dt = -\frac{dH}{H-y_o} \qquad /11/$$

The solution of equation /11/

$$\frac{A}{\beta}t = -\ln(H-y_o)+C \qquad /12/$$

The boundary condition t=o, $H=H_0$, and thus $C=\ln /H_0-y_0/$.

By substituting these terms into equation /12/

$$H-y_o = \frac{H_o-y_o}{\exp\left(\frac{A}{\beta}t\right)} \qquad /13/$$

Eq /13/ situated into Dupuit-Thiem expression, we get the function Q=f/t/.

$$Q = \frac{2\pi mk(H_o-y_o)}{\ln R_o/r}\cdot \exp\left(-\frac{A}{\beta}t\right) \qquad /14/$$

4./ Calculation of Drawdown of Well Situated into Finite Aquifer with Constant-discharge Condition.

As the drawdown radius /R/ attained the boundary of the aquifer, and constant discharge is requested, in this case $H-y_o$ has to remain constant.

With notations according to Fig /3/

$$\beta R_o^2 \pi ds = Q_o dt \qquad /15/$$

Evaluating the integral

$$\beta R_o^2 \pi s = Q_o t + C \qquad /16/$$

If $t=o$, $s=H-y_o=s_o$; $c=\beta R_o^2 \pi s_o$ /17/

By substituting these terms into equation /16/ we obtain

$$s = s_o + \frac{Q_o t}{\beta R_o^2 \pi} \qquad /18/$$

In Eq /18/ the value of t is to be counted, when R attained the boundary of the aquifer.

5./ Attaining Time /t_v/ of the Boundary of the Aquifer /$R=R_o$/

From Eq /5/ t may be expected, and if $R=R_o$ thus $t=t_v$, ie:

$$t_v = R_o^2\left(\ln\frac{R_o}{r} - \frac{1}{2}\right)\frac{\beta}{2mk} + \frac{\beta r^2}{4mk} \qquad /19/$$

neglecting $\frac{\beta r^2}{4mk}$ (β is usually small)

$$t_v = R_o^2\left(\ln\frac{R_o}{r} - \frac{1}{2}\right)\frac{\beta}{2mk} \qquad /20/$$

6./ Examples

Consider the aquifer with parameters indicated in Fig /4/, with R_o=20.000 m H=160 m, m=60 m, k=8 m/day, β =0,0001 and r=0,1 m. The drawdown will be constant s_o=5 m.

Of interest is the funcion of discharge Q=f/t/.

At first t_v is computed from Eq /20/ /Eqs /5/, /6/ are valid till $t<t_v$/.

$$t_v = 20000^2\left(\ln\frac{20000}{0,1} - \frac{1}{2}\right)\frac{0,0001}{2\cdot 60\cdot 8} = 487 \text{ days}$$

R can be calculated from Eq /5/. For example t=10 days. For the computation let us take R_1=2500 m, thus

$$R = \sqrt{\frac{\frac{2\cdot 60\cdot 8}{0,0001}10 - \frac{0,1^2}{4}}{\ln\frac{2500}{0,1} - \frac{1}{2}}} = 3157 \text{ m}$$

3157 m is substituted into the right-hand side of Eq /5/ i.e. in ln R. Than we get 3122 m. That is acceptable.

The discharge of well 10 days after pumping started /with Eq 6./

$$Q = \frac{2\pi\, 60 \cdot 8\,(60-55)}{\ln \frac{3122}{0,1}} = 1457\ m^3/d$$

In Fig./5/ the discharge hydrograph constructed from the other data is also given.

But the validity of this curve is thus restricted to 487 days. After 487 days of pumping started the discharge /Q/ was computed with Wq /14/. In this equation the value of time /t/ begins at 487 days. These parts of the curve were drawn in Fig. /5/ too.

Consider the aquifer formation as above and with the same data, but there will be constant discharge Q_1=750 m/day. Compute the drawdown curve profile in the period s=f/t/.

Till t_v=487 days the equation /7/ will be valid, calculating with the know R /with Eq 5/. After 487 days the Eq /18/ is used.

The results of the calculation can be seen in Fig /6/. The drawdown sinks rapidly after t_v /487 days/.

7./ Determination β Factor.

β can be determined by test pumping, according to Eq /21/. Eq /21/ is derivoted from Eq /5/, i.e:

$$\beta = \frac{2mkt}{\frac{r^2}{2} + R^2\left(\ln\frac{R}{r} - \frac{1}{2}\right)} \qquad /21/$$

The basic data of the aquifer are known. /m,k/ Using Eq /21/ it is necessary to have a pumped well and an observed well. The distance betwen two wells is R_1 and the time of beginning of drawdown at the observed well after pumping started /t_1/ has to be measured. These values /R_1 t_1/ β can be calculated.

Examplo

Et Fonyód two wells were situated. The distance of two wells is R_1=360 m. The parametors of the aquifer: m=11 m, k=6,2 m, /fine sand/ Pumping from well I. /r=0,1 m/, sinking of water level at well II. began t_v=2 hours=0,083 day after pumping started.

According to Eq /18/.

$$\beta = \frac{2 \cdot 11 \cdot 6{,}2 \cdot 0{,}083}{360^2\left(\ln\frac{360}{0{,}1} - \frac{1}{2}\right)} = 0{,}0000114$$

Et Eq /21/ $\frac{r^2}{2}$ can be negligated.

8./ Test of Vertification of the Method

At water research for water-work system of County Győr-Sopron a testing well group was built. Beside a lot of question /for example testing of Hantush's model/ we examined the vertification of above calculating model.

The testing aquifer is caraoterized by $k=5$ m/day /fine sand/, $m=13$ m, $\beta=0{,}000012$. Radius of the wells are $r=0{,}15$ m.

Pumping test is made on one of them with variable constant discharge. The measured and calculated drawdown curves were drown in Fig./7/. It can be seen, that the measured and the calculated drawdowns are practically same.

II. OPERATING OF WELL GROUPS.

1./ Calculating of Drawdowns of two wells Constant-discharge Condition.

At these cases the superposition of velocity potential is used.

As khown the velocity potential:

$$\phi = -kh$$

where h is the piezometrical level at the observed print.

At an artesian well

$$\phi_1 = -\frac{Q_1}{2\pi m}\ln r_1 + C$$

where r_1 the distance from the well to the observation point.

At two artesian wells the total potential are: /see Fig.8./

$$\phi = \phi_1 + \phi_2$$

i.e:

$$-kh = -\frac{Q_1}{2\pi m}\ln r_1 - \frac{Q_2}{2\pi m}\ln r_2 + C \qquad /22/$$

value of C can be calculated from boundary conditions according to Fig./9/, thus

$$H-h_1 = s_1 = \frac{Q_1}{2\pi mk}\ln\frac{R_1}{r_{10}} + \frac{Q_2}{2\pi mk}\ln\frac{R_2}{r_{10}} \qquad /23/$$

where s_1 = the drawdown of well 1.
Q_1 = discharge of well 1.
r_{10} = radius of well 1.
b = distance between the two wells
Q_2 = discharge of well 2.

It $Q_1=Q_2=Q$; $R_1=R_2=R$, thus

$$s_1 = \frac{Q}{2\pi mk}\ln\frac{R}{r_{10}b} \qquad /24/$$

Calculating R from Eq /5/, its value can be substituted Eqs /23,24./ and function $s_1=f/t/$ can be calculated. Using Eqs /24, 23./ it must be, that $b < R$.

The drawdown of well 2 is:

$$s_2 = \frac{Q_1}{2\pi mk} \ln \frac{R_1}{b} + \frac{Q_2}{2\pi mk} \ln \frac{R_2}{r_{20}} \qquad /25/$$

where r_{20}= the radius of well 2.

If $Q_1=Q_2=Q$; and $R_1=R_2=R$, thus

$$s_2 = \frac{Q}{2\pi mk} \ln \frac{R^2}{b\, r_{20}} \qquad /26/$$

2./ Drawdowns at Three Operating wells with Constant-Discharge Condition.

Drawdown as function of pumping time can be deriveted as in cases of two wells.

The drawdown at well 1.

$$s_1 = \frac{Q_1}{2\pi mk} \ln \frac{R_1}{r_{10}} + \frac{Q_1}{2\pi mk} \ln \frac{R_2}{b_{12}} + \frac{Q_3}{2\pi mk} \ln \frac{R_3}{b_{13}} \qquad /27/$$

where b_{12} = dictance betwen well 1 and well 2.
b_{13} = dictance betwen well 1 and well 3.

if $Q_1=Q_2=Q_3=Q$, and $R_1 \cong R_2 \cong R_3 \cong R$

$$s_1 = \frac{Q}{2\pi mk} \ln \frac{R^3}{r_{10}\, b_{12}\, b_{13}} \qquad /28/$$

The drawdown at well 2

$$s_2 = \frac{Q_1}{2\pi mk} \ln \frac{R_1}{b_{12}} + \frac{Q_2}{2\pi mk} \ln \frac{R_2}{r_{20}} + \frac{Q_3}{2\pi mk} \ln \frac{R_3}{b_{23}} \qquad /29/$$

with simplified condition

$$s_2 = \frac{Q}{2\pi mk} \ln \frac{R^3}{r_{20}\, b_{12}\, b_{23}} \qquad /30/$$

The drawdown at vell 3:

$$s_3 = \frac{Q_1}{2\pi mk} \ln \frac{R_1}{b_{23}} + \frac{Q_2}{2\pi mk} \ln \frac{R_2}{b_{23}} + \frac{Q_3}{2\pi mk} \ln \frac{R_3}{r_{30}} \qquad /31/$$

and the simplified equation is

$$s_3 = \frac{Q}{2\pi mk} \ln \frac{R^3}{r_{30}\, b_{23}\, b_{13}} \qquad /32/$$

3./ Calculating of Drawdowns by Operating Wells, Number n.

If we have wells number n, the drawdown at the well i can be derived at the same above.

The simplified equation is:

$$s_i = \frac{Q}{2\pi mk} \ln \frac{R^n}{r_{i0}\, b_{1i}\, b_{2i} \cdots b_{ni}} \qquad /33/$$

Where r_{io} = radius of well i
b_{1i} = distance betwenn well i and well 1.
b_{2i} = distance between well i and well 2.
With the above equations can be used if $R > b_{in}$, and $R < R_o$.

5./ Examples

At waterworks Fonyód the aquifer can be caracterized by following data:
thickness of aquifer m=11 m, k=6,2 m/day /fine sand/, β =0,0000114, H=100 m.

It is desirable to get 900 l/min. We get only 300 l/min. from one well, for that reason it needs three wells, with r=0,1 m radius. The distance between wells will be 360 m. We want to determine the drawdowns depending of pumping time. In our case $Q_1=Q_2=Q_3=$ 300 l/min=432 m^3/d. $R_1=R_2=R_3=R$; $r_{10}=r_{20}=r_{30}$; we calculate with the simplified equation.

First we calculated R, depending on time /t/ with Eq /5/, and drawdowns with Eq /32/.

The calculated values of R and s, can be seen in Table 1.

The smaller the distance between the wells, the deeper the drawdowns will be.

If the distance between the wells /b/ are smaller than 360 m, i.e b=50 m, at t=365 days, s will be 24,1 m instead of 20,4 m /three wells are operating/.

6./ Discharge of Well Groups with Constant-drawdown Conditions.

At constant drawdown it is desirable to get the discharge function depending on pumping time, i.e Q_1 and Q_2, have to be calculated, from Eqs /23,25./ in the case of two wells. Q_1 and Q_2 can be expressed and calculated, as s_1 and s_2 are known.

If $Q_1=Q_2=Q$; $R_1=R_2=R$; $s_1=s_2=s_o$; $r_{20}=r_{10}=r_o$, from Eq /26/:

$$Q = \frac{s_o \, 2\pi m k}{\ln \frac{R^2}{r_o b}} \qquad /34/$$

If we have three wells, the simplified equation is:

$$Q = \frac{s_o \, 2\pi m k}{\ln \frac{R^3}{r_o \, b_{12} \, b_{13}}} \qquad /35/$$

If the number of the wells is "n", the discharge of well i can be derived as seen above.

The simplified equation is:

$$Q = \frac{s_i \, 2\pi m k}{\ln \frac{R}{s_i \, b_{1i} \, b_{2i} \cdots b_{ni}}} \qquad /36/$$

The Eqs /34,35,36/ are valid if $R < R_0$ and $R > b$.

Example: The discharge of three wells at Fonyód are to be calculated, at $s_1=s_2=s_3=s_0=6$ m. Data: $b_{12}=b_{13}=b=360$ m. The calculations were made as shown above.

The results of the calculation ca be seen in Table 2. /Values of R/t/ in Table 1./

B./ DEWATERING OF WATER-TABLE AQUIFER WITH CONSTANT-DISCHARGE CONDITION

As an example a pair of formulas is presented which can be used for the calculation of necessary data of mine drainage in a first approximation. /There is no recharge/. These are the function of depression s=f/t/

$$s = H - \sqrt{H^2 - \frac{1}{\pi k} Q \ln \frac{R}{r}} \qquad /37/$$

the function of the depression range R=f/t/

$$R = \sqrt{\frac{k\,(2H-s)\,t}{\mu\alpha\left[\ln\frac{R}{r} - \frac{1}{2}\right]} - \frac{r^2}{2\left[\ln\frac{R}{r} - \frac{1}{2}\right]}} \qquad /38/$$

where: k is the coefficient of permeability of Darcy, μ is the free, stress-free, gravitational porosity of the rocks, r is the radius of the well, H is the original head is the time of pumping, α is a shape or correction factor determined by experiments based on Eq /38/

The procedure of calculation is then as follows:

A Q value is estimated in order to calculate the function

$$s = f/t/.$$

Hereafter an s_1 value is taken and is introduced into Eq. /38/. Then an R_1 value is estimated to pumping time t_1 and is substituted in to the right-hand side of Eq./38/, i.e. in ln R. With these R_1 and s_1 value R_2 is calculated on the left-hand side of Eq./38/. R_2 is usually not equal to R_1. Therefore, R_2 is entered at the right-hand side of Eq./38/ and a value R_3 is calculated. This procedure must be continued until the estimated and the calculated R-s coincide. /This will usually happen after 3-4 approcimations./

According to the above scheme an R value is calculated belonging to pumping time t_1 and depression s_1. This is now entered in to Eq./37/ and the value of s_2 will be obtained.

If this is equal to the value of s_1 introduced in Eq /38/ originally the result is good. If not, then the s_2 value obtained must be reintroduced in Eq /38/ and the whole iteration must be repeated until the estimated and calculated values become equal.

From the character of the calculation it follows that the problems fits the computers and due to the great number of repetitions it can be solved only by computers.

Example:

There is a mine having an area a radius $r=3000$ m. Here the level of the karstic water must be lowered to $s=116$ m. The present level of karstic water is $H=141$ m. The characteristics of the karstic rock are: $k=35$ m/day, $\mu=0{,}01$, $\alpha=0{,}3$.

If $Q=300.000$ m^3/day is pumped steadily, then according to Fig.11. the depression after 15 years /5475 days/ is only $s=35$ m, despite the fact that the range reaches 72,987 m or round 73 kms. This pumping rate is actually insufficient to dewater to mine.

Taken $Q=600.000$ m^3/day, again from Fig.11. it is apparent that the depression is 86 m after 15 years.

If $Q=800.000$ m^3/day, the desired depression $s=116$ m can be reached in appr. 2.200 days /6years/.

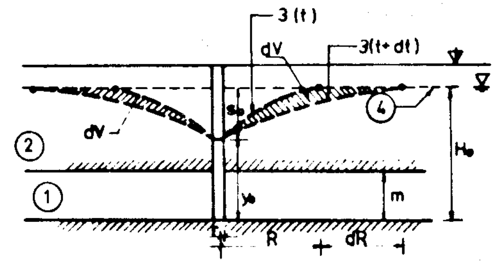
Table 1

time after pumping started days	R m	operating 1 well s_1 m	operating 2 wells s_1 m	operating 3 wells s_1 m
1	1.130	9,35	10,46	11,59
10	3.900	10,60	12,97	15,34
30	5.700	11,00	13,76	16,52
100	10.200	11,54	14,88	18,22
200	14.100	11,85	15,51	19,17
365	19.000	12,20	16,20	20,40
730	26.500	12,48	16,78	21,08
1825	41.000	12,90	17,63	22,36

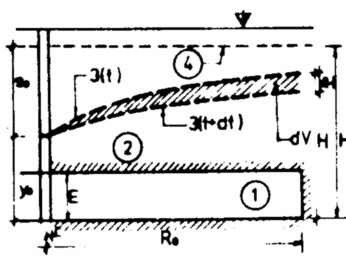
Table 2.

time after pumping started days	yield of each well m^3/day
1	203
10	153
30	142
100	129
200	122
365	117
730	111
1825	105

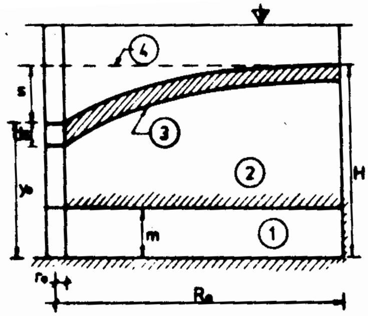
List of Figures



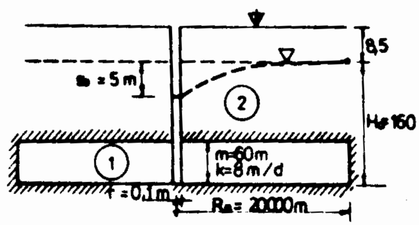
1.ábra Fig.1.



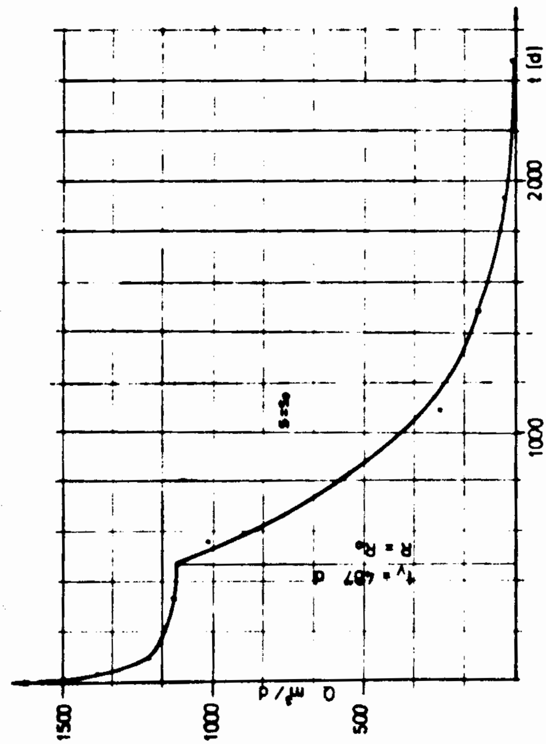
2.ábra Fig. 2.



3.ábra Fig. 3.



4.ábra Fig. 4.



5.ábra

Fig. 5.

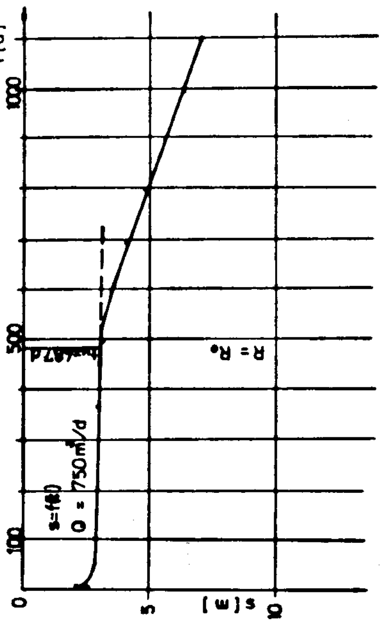
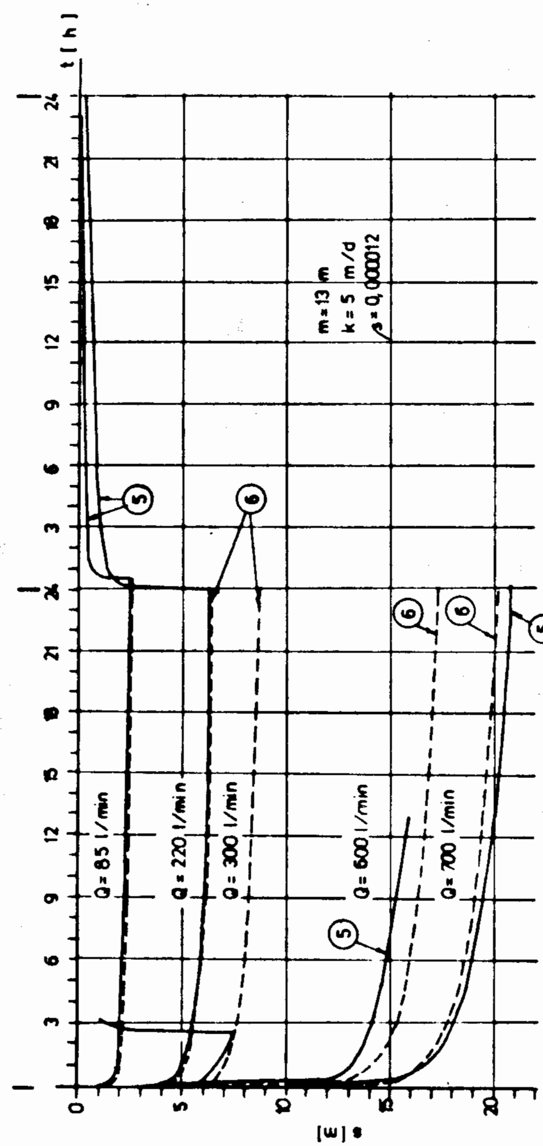


Fig. 6.

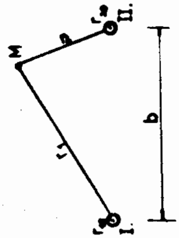
6.6bra



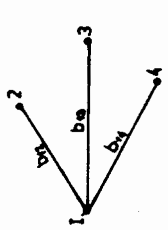
t [h]
s [m]
0
5
10
15
20
3
6
9
12
15
18
21
24
Q = 85 l/min
Q = 220 l/min
Q = 300 l/min
Q = 600 l/min
Q = 700 l/min
5
6
m = 13 m
k = 5 m/d
β = 0,000012

Fig.7.

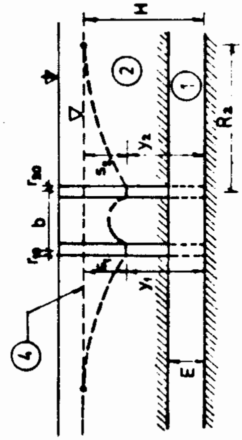
7. ábra



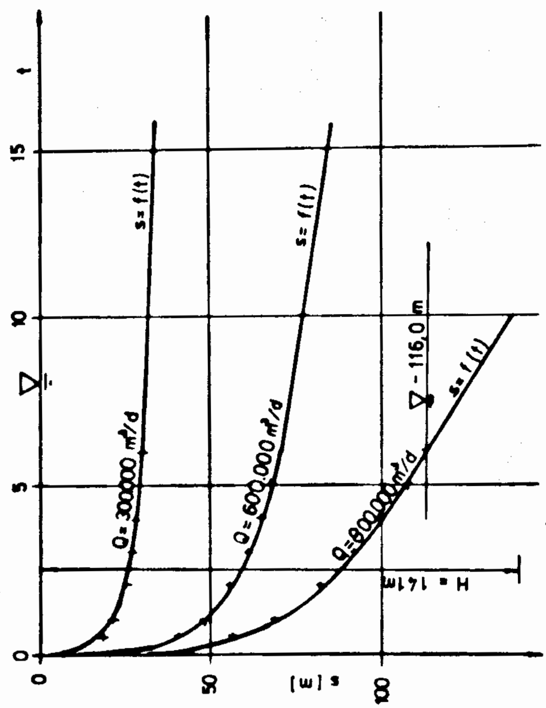
8. ábra Fig. 8.



10 ábra Fig. 10.



9. ábra Fig. 9.



11.ábra

Fig. 11.